応用数学 A

積分定理

$$div\,F = 0?\quad \neq 0?$$

$$rot\,F = 0?\quad \neq 0?$$

積分定理　　おさらい

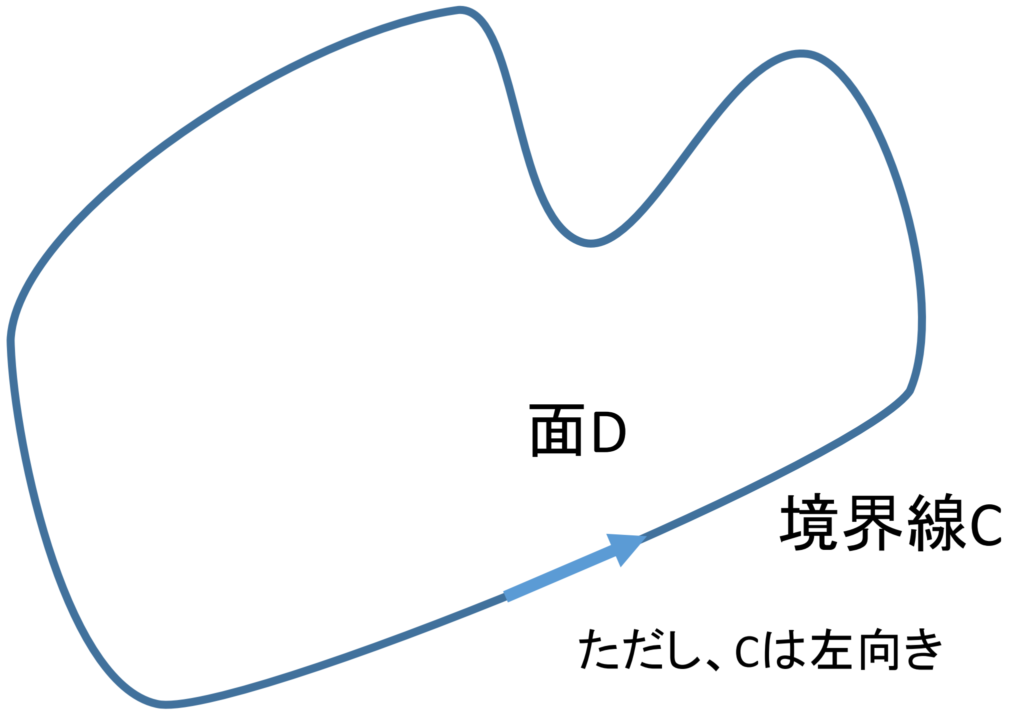
Gaussの定理 $\int divBdV = \int B \cdot ndS$

Stokesの定理 $\int \nabla \times E \cdot bdS = \int E \cdot dr$

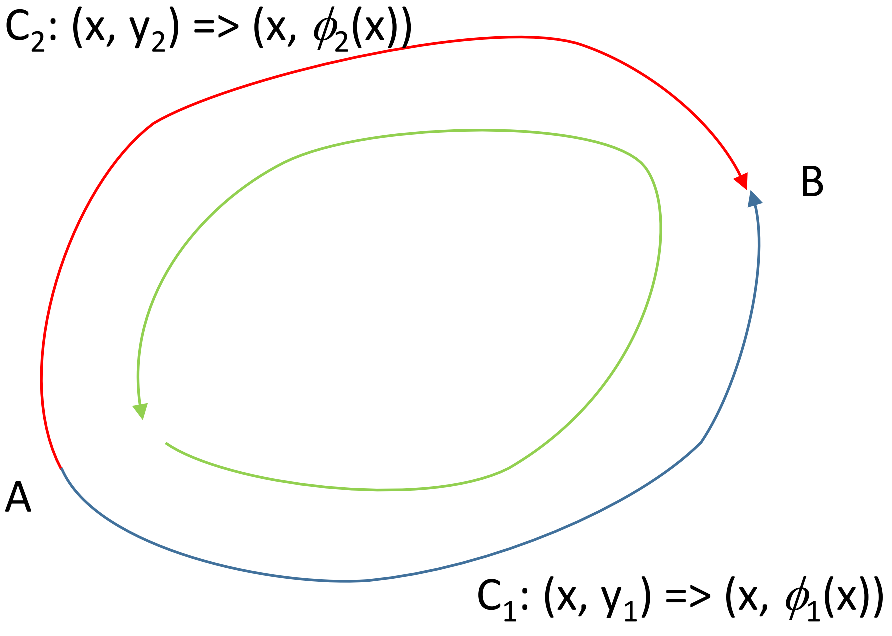
Greenの定理 $\iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dxdy = \int_C (fdx + gdy) = \int_C \left(f\vec{e_i} + g\vec{e_j}\right) \cdot \overrightarrow{dr}$

Gaussの発散定理 $\int_S n \times FdS = \int_V \nabla \times FdV$

Green　の　定理



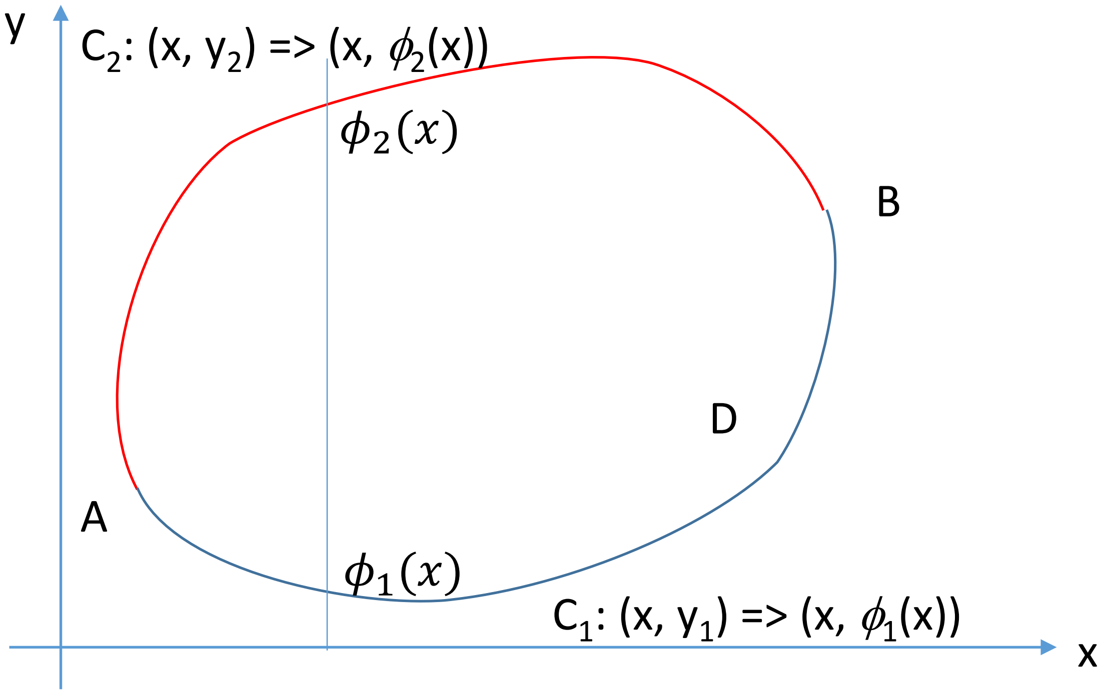
$$\iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dxdy = \int_C \left(f dx + g dy\right) = \int_C \left(f\vec{e_i} + g\vec{e_j}\right) \cdot \overrightarrow{dr}$$



$$\int_C f\,dx = \int_{C1-C2} f\,dx = \int_{C1} f\,dx - \int_{C2} f\,dx$$

$$= \int_A^B f(x, y_1(x))dx - \int_A^B f(x, y_2)(x)dx$$

$$= \int_A^B f(x, \phi_1(x))dx - \int_A^B f(x, \phi_2(x))dx$$



$$\iint_D \left(\frac{\partial f}{\partial y}\right) dxdy = \int_A^B \left(\int_{\phi_1(x)}^{\phi_2(x)} \frac{\partial f}{\partial y} dy\right) dx = \int_A^B [f]_{\phi_1(x)}^{\phi_2(x)} dx$$

$$= \int_A^B (f(x, \phi_2(x)) - f(x, \phi_1(x)))\,dx$$

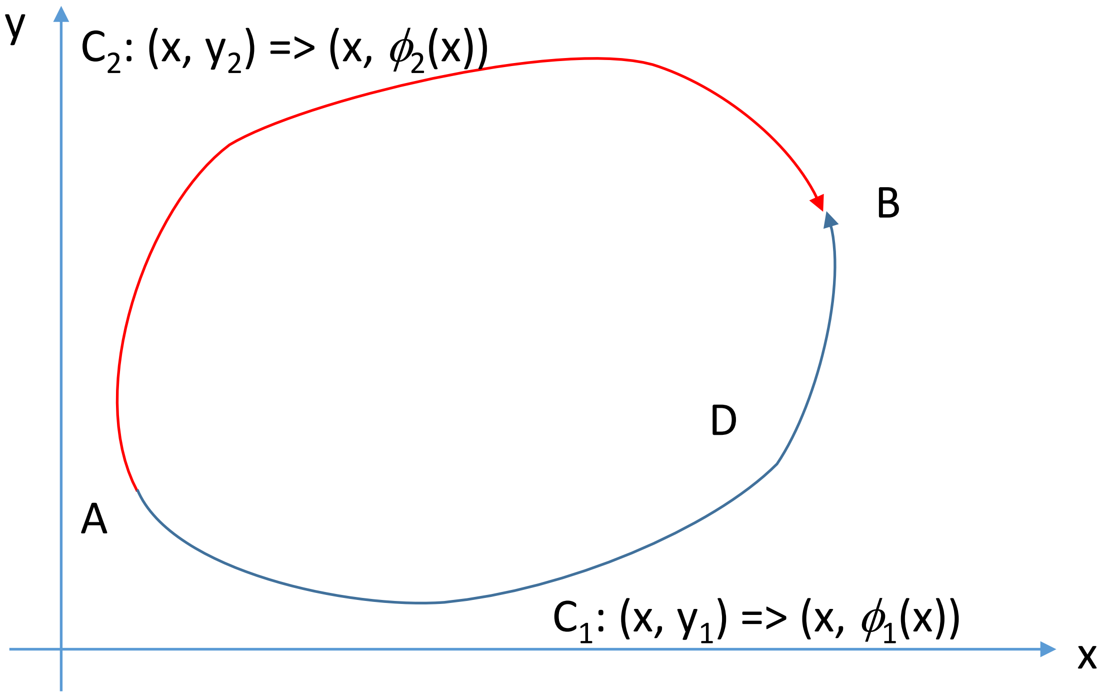
$$= \int_A^B f\big(x, \phi_2(x)\big)dx - \int_A^B f\big(x, \phi_1(x)\big)dx$$

$$\int_C f\,dx = -\iint_D \left(\frac{\partial f}{\partial y}\right) dxdy$$

$$\int_C g\,dy = -\iint_D \left(\frac{\partial g}{\partial x}\right) dxdy$$

Greenの定理

$$\int_C f\,dx + g\,dy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dxdy$$



$$\int_C fdx + gdy = \iint_D \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dxdy$$

$$\int_C fdx + gdy = \int_{C1} fdx + gdy - \int_{C2} fdx + gdy$$

もし $\dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y} = 0$ ならば

$fdx + gdy$ は経路によらずに同じ値をとる。

＝＞ポテンシャルみたいなもの＝＞Ψ(x,y)

$$\int_{C1,C2} fdx + gdy = \Psi(B) - \Psi(A)$$

ここで、 $d\Psi = \dfrac{\partial \Psi}{\partial x} dx + \dfrac{\partial \Psi}{\partial y} dy$

$$\Psi(B) - \Psi(A) = \int_A^B d\Psi = \int_A^B \frac{\partial \Psi}{\partial x} dx + \frac{\partial \Psi}{\partial y} dy$$

$\dfrac{\partial \Psi}{\partial x} \to f$, $\dfrac{\partial \Psi}{\partial y} \to g$

$$div\, F = 0? \quad \neq 0?$$

$$rot\, F = 0? \quad \neq 0?$$

Gaussの定理 $\int divBdV = \int B \cdot ndS$

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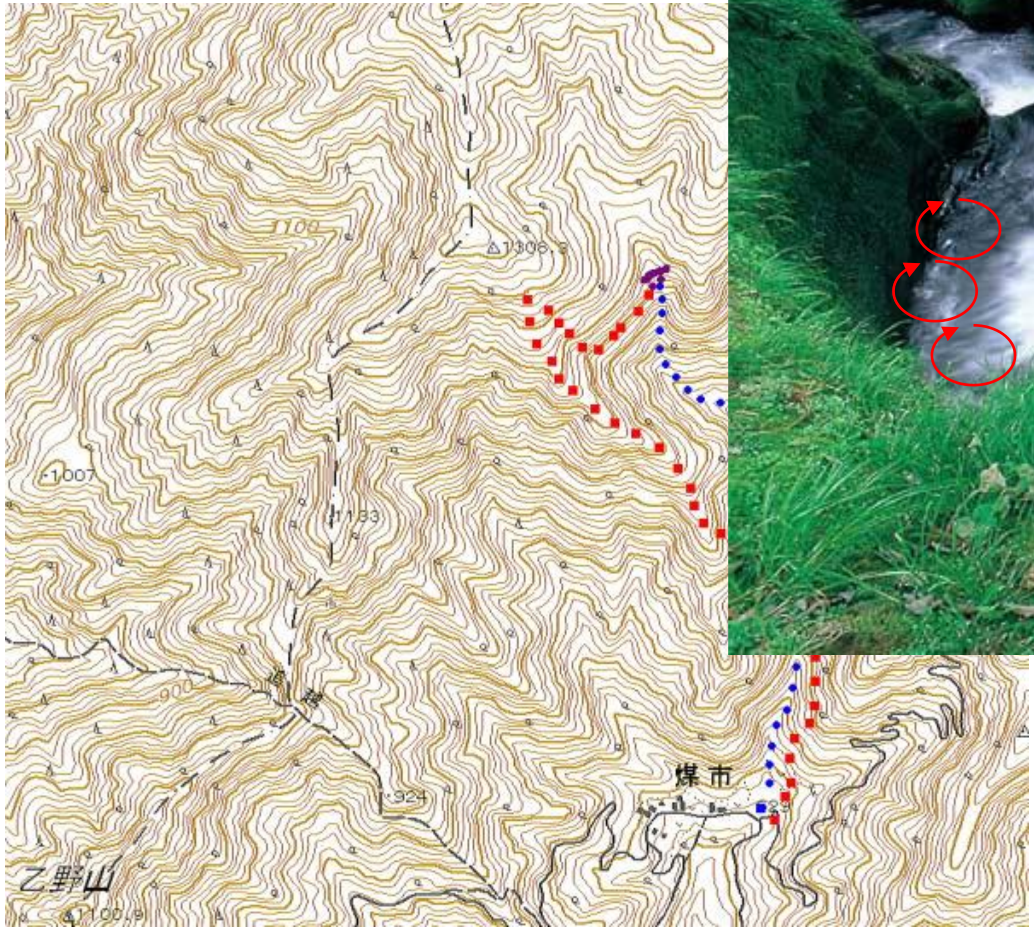
Gaussの発散定理 $\int_S n \times FdS = \int_V \nabla \times FdV$

ベクトル演算

gradient $\nabla \phi$

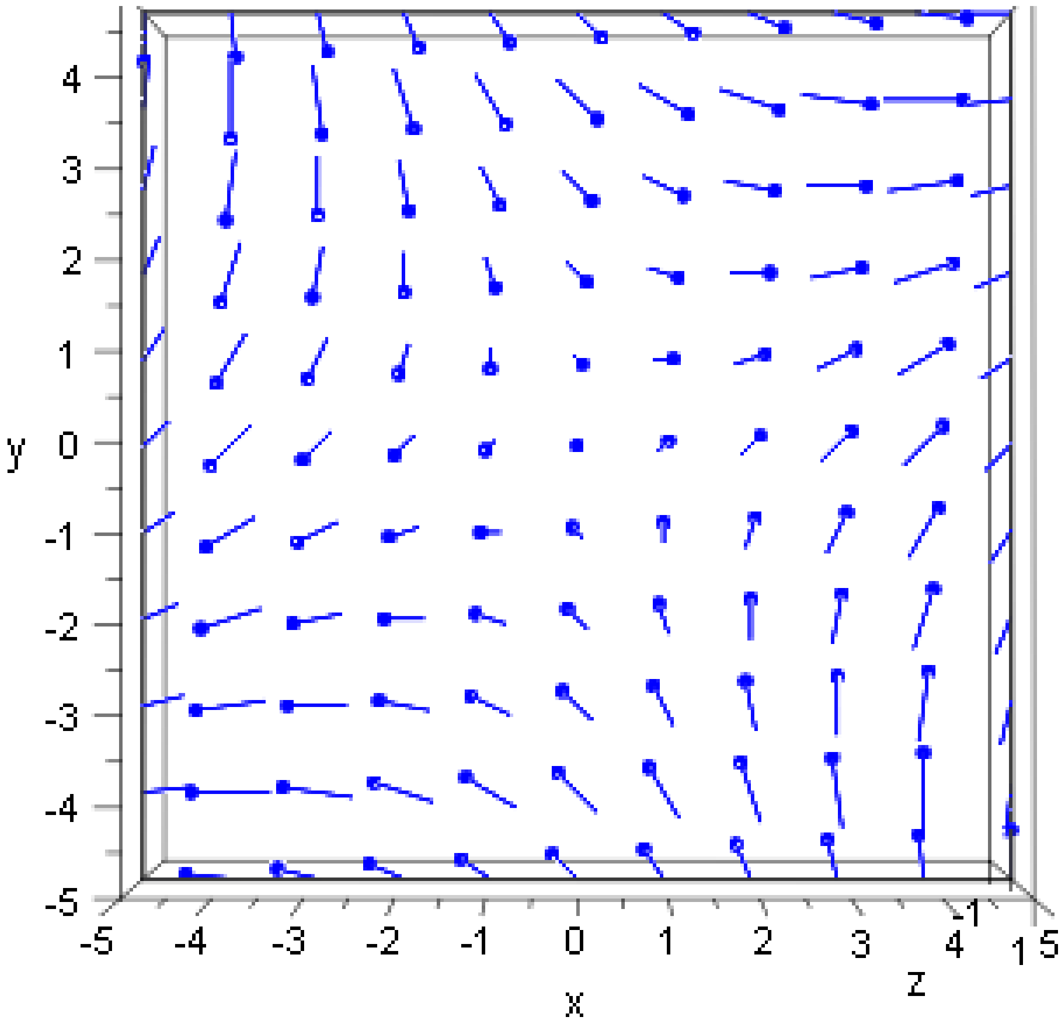
divergence $\nabla \cdot \phi$

rotation $\nabla \times \phi$



$$div\, F = 0? \quad \neq 0? \quad rot\, F = 0? \quad \neq 0?$$

$$\vec{F} = \begin{pmatrix} fx \\ fy \\ fz \end{pmatrix}$$

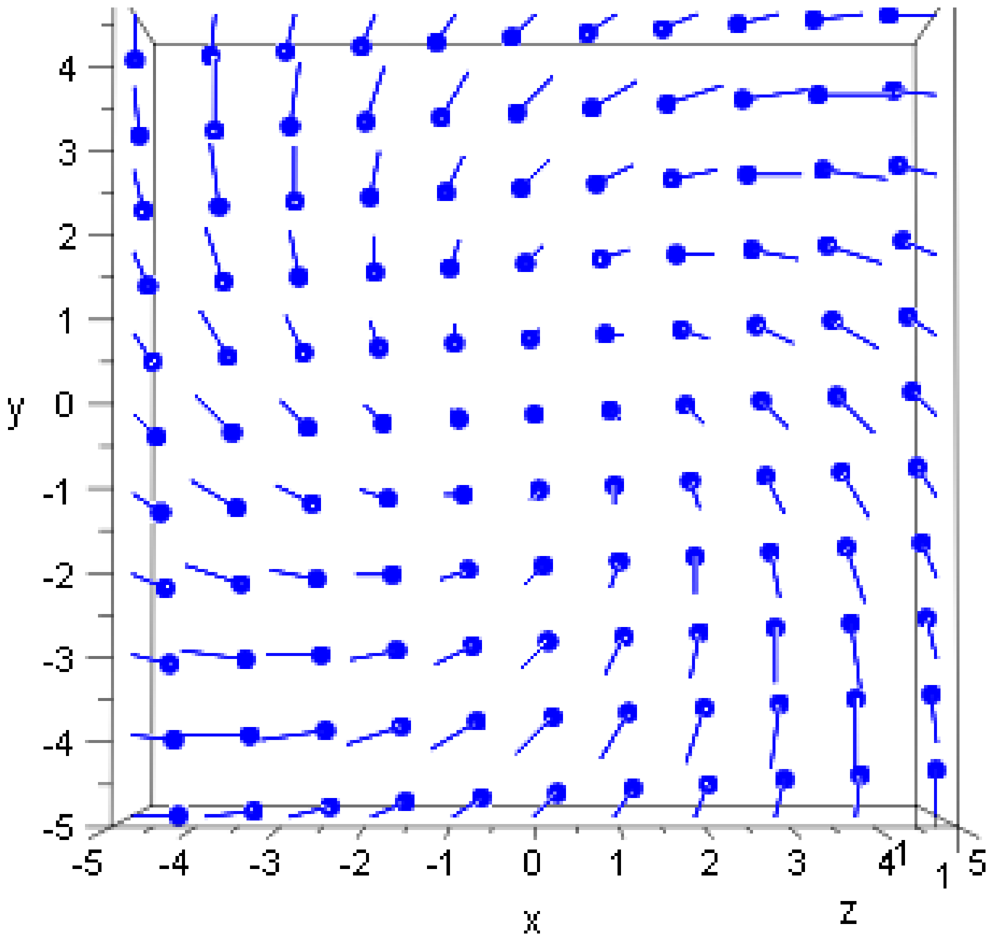


$$fx := x + y$$
$$fy = x - y$$
$$fz = 0$$

$$div\, F = 0$$

$$rot\, F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$div\ F = 0? \quad \neq 0? \quad rot\ F = 0? \quad \neq 0?$$

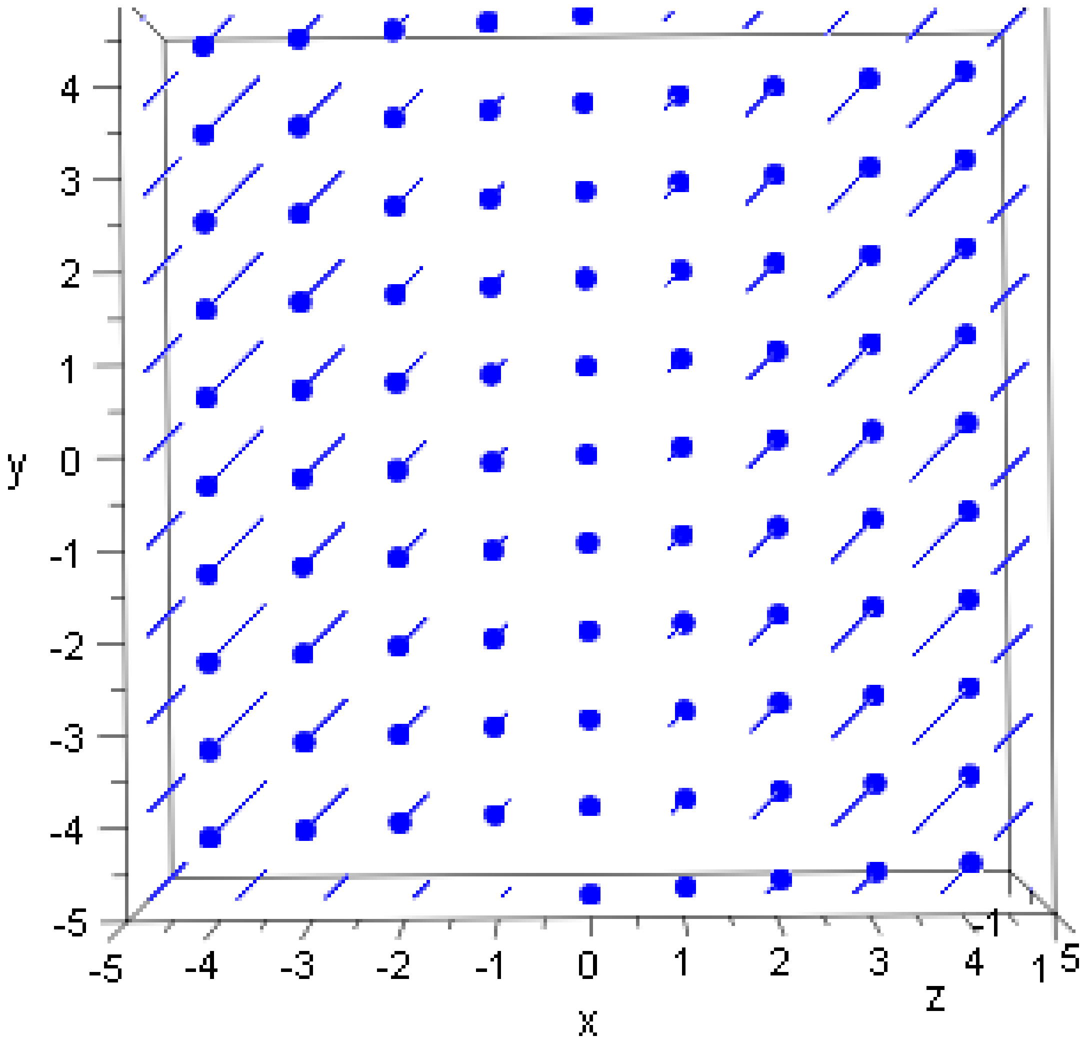


$$fx := -x - y$$
$$fy = x - y$$
$$fz = 0$$

$$div\ F = -2$$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$div\, F = 0? \quad \neq 0? \quad rot\, F = 0? \quad \neq 0?$



$fx := x$

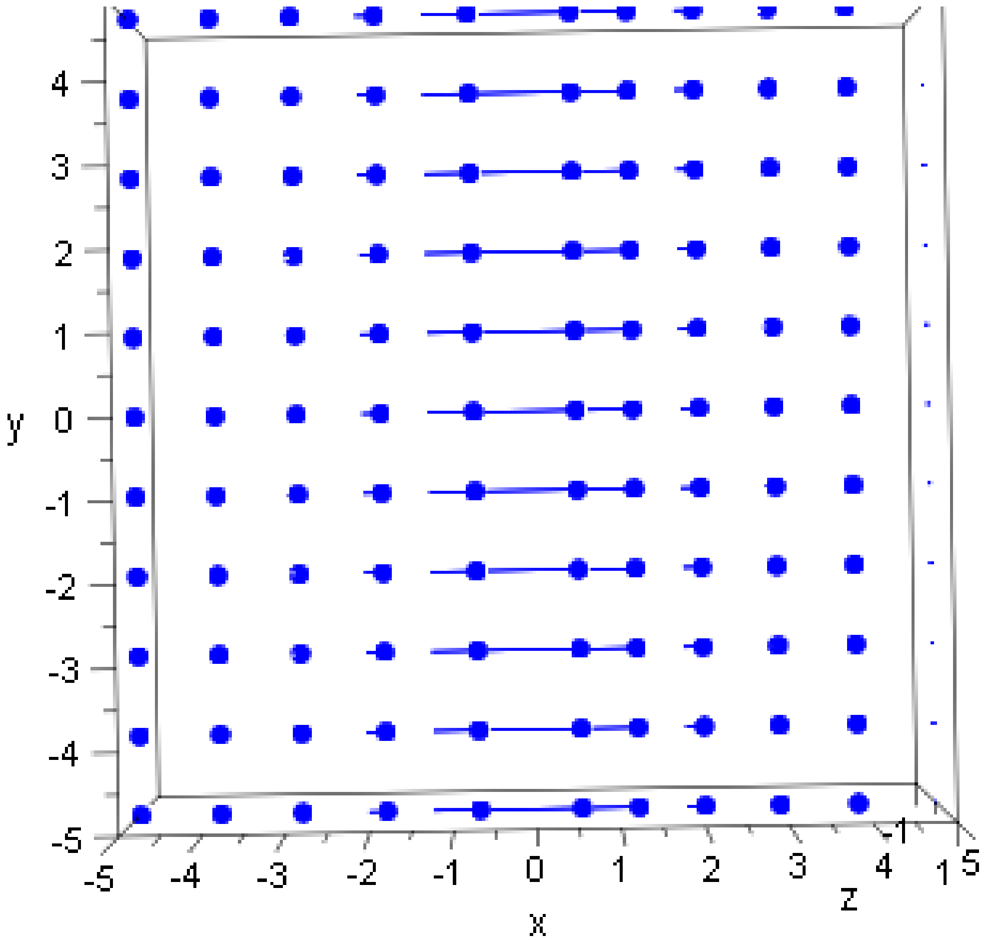
$fy = x$

$fz = 0$

$div\, F = 1$

$$rot\, F = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$div\, F = 0? \quad \neq 0? \quad rot\, F = 0? \quad \neq 0?$$



$$fx := \frac{1}{1+x^2}$$

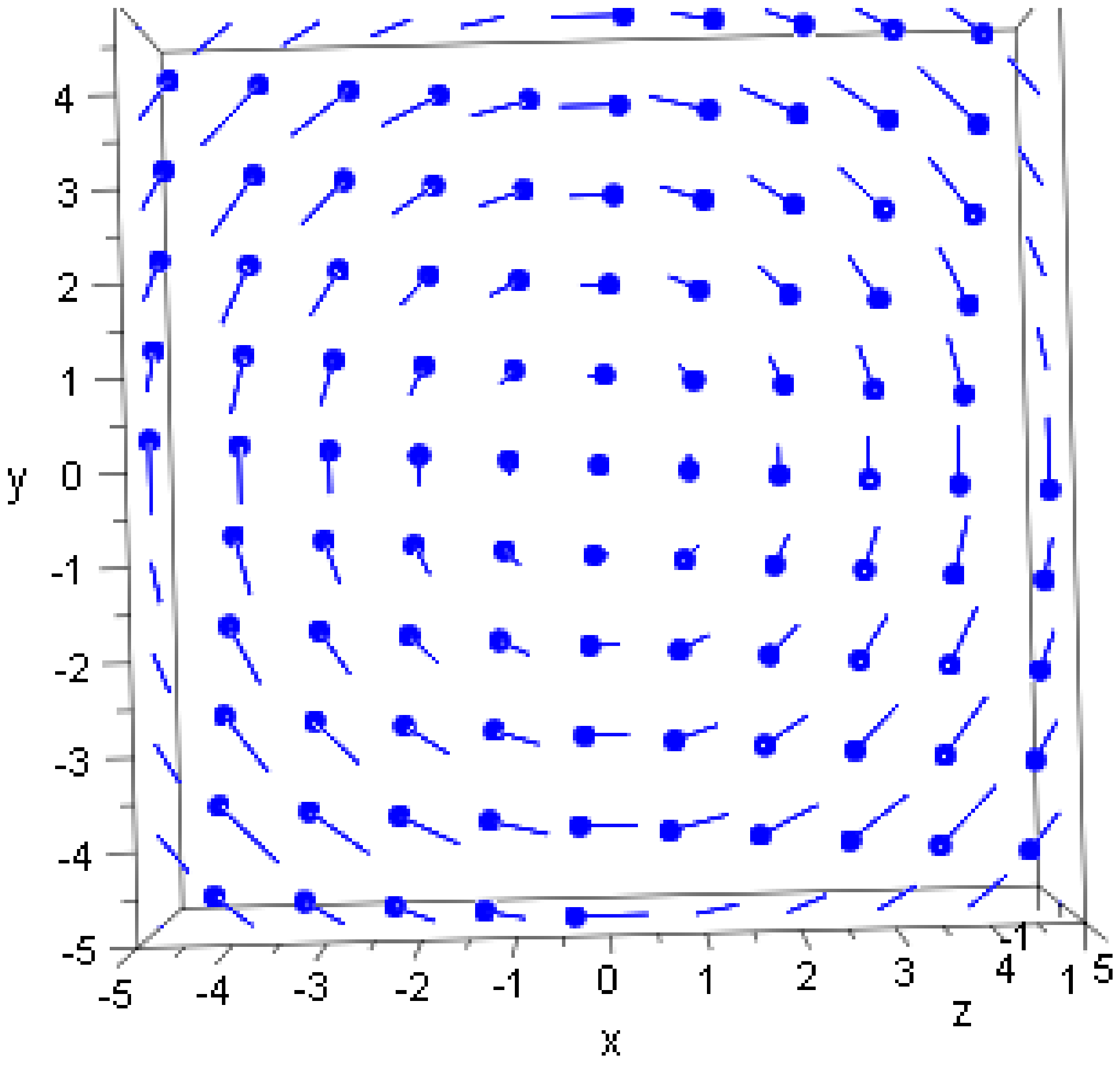
$$fy = 0$$

$$fz = 0$$

$$div\, F = \frac{-2x}{(1+x^2)^2}$$

$$rot\, F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$div\ F = 0? \ \neq 0? \ \ rot\ F = 0? \ \neq 0?$



$fx := y$

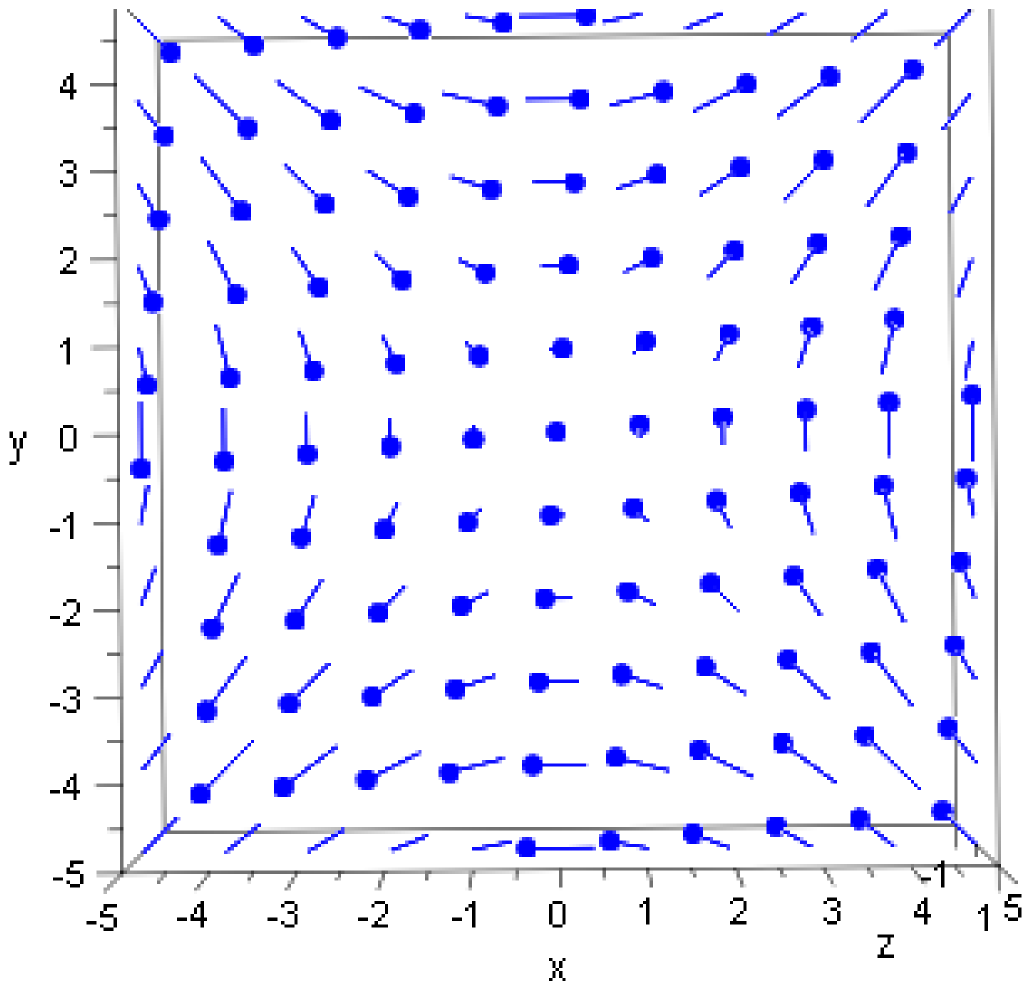
fy: =-x

$fz = 0$

$div\ F = 0$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$div\ F = 0?\ \ \neq 0?\ \ rot\ F = 0?\ \ \neq 0?$

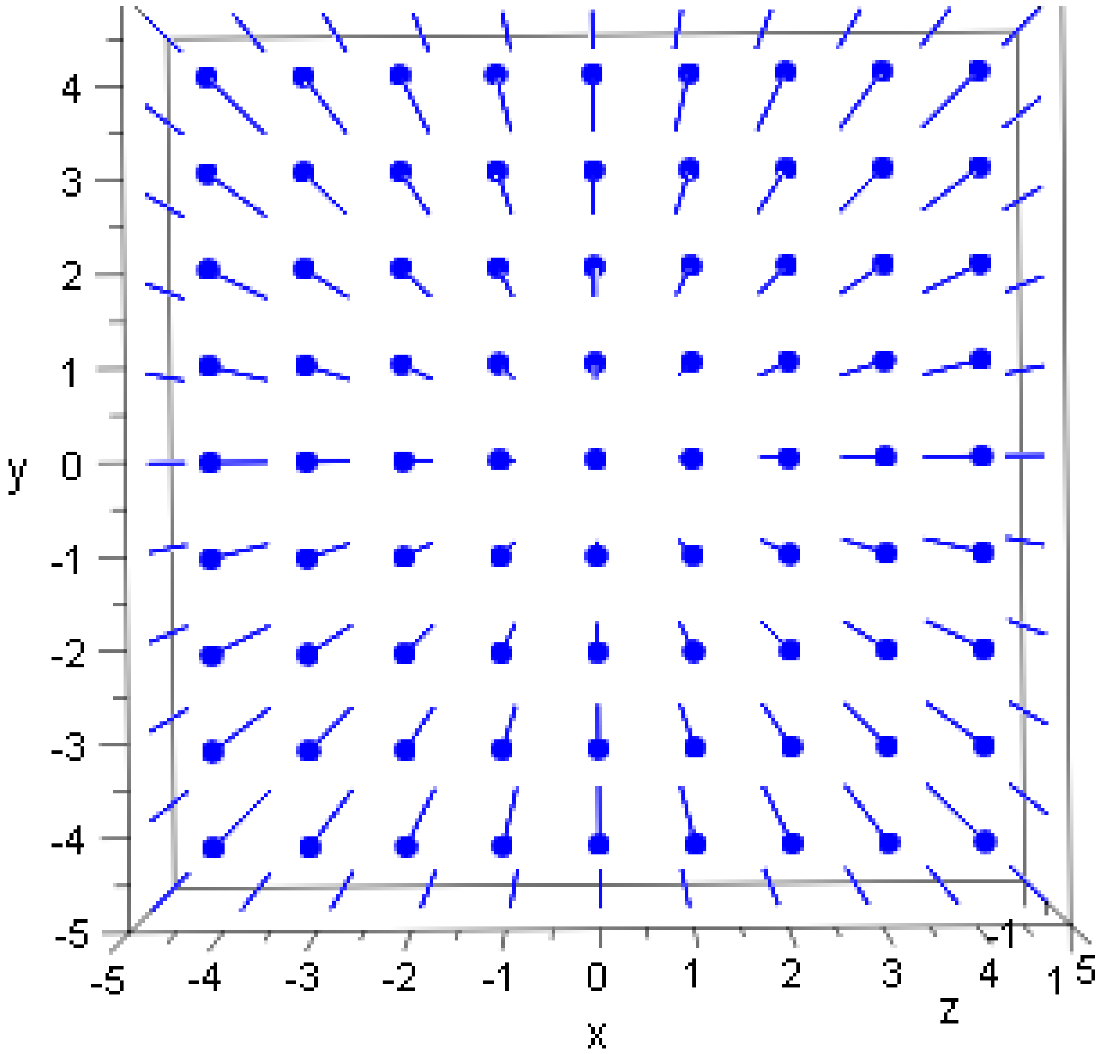


$fx := y$
$fy := x$
$fz = 0$

$div\ F = 0$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$div\ F = 0? \quad \neq 0? \quad rot\ F = 0? \quad \neq 0?$$

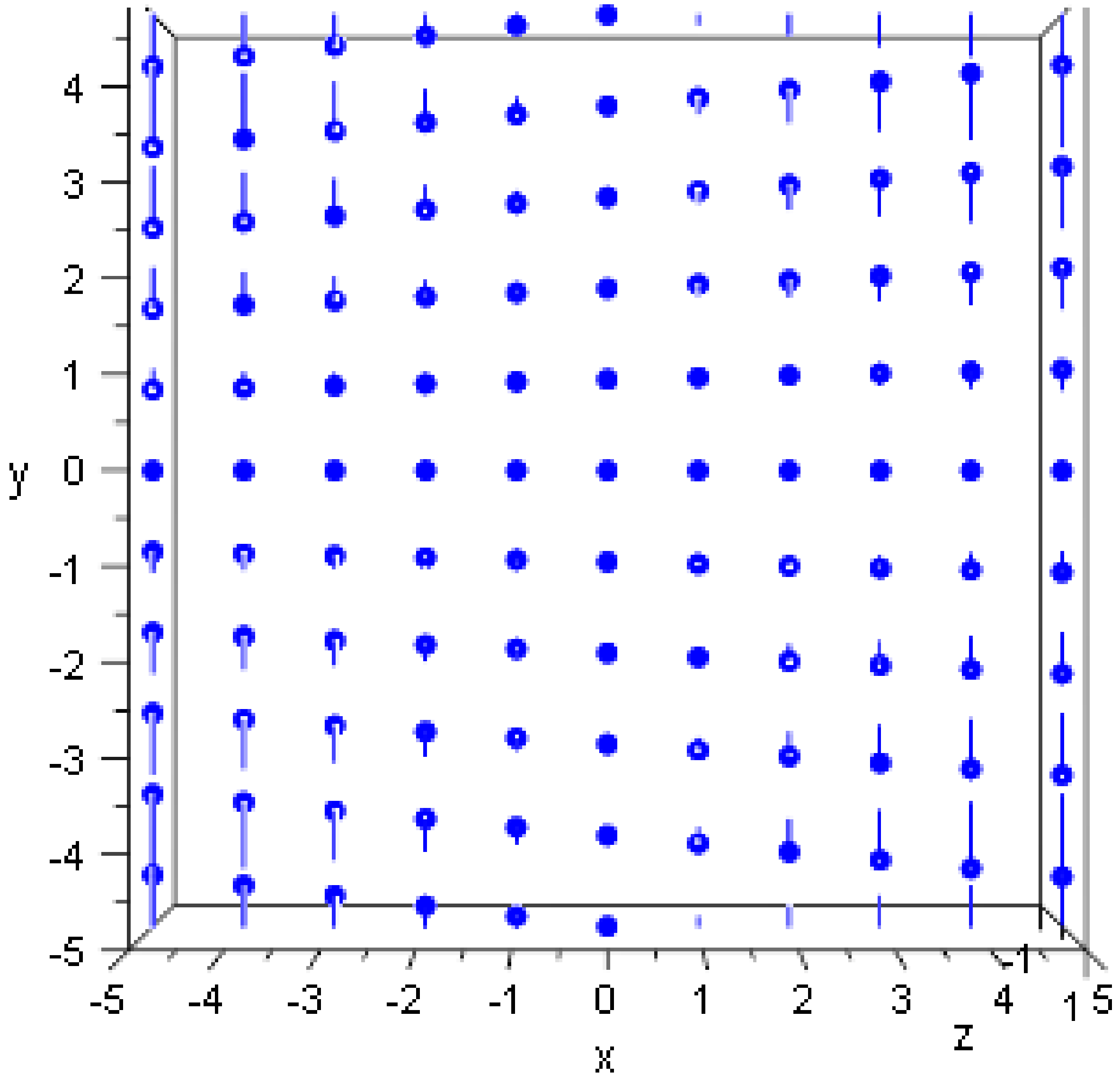


$$fx := x$$
$$fy := y$$
$$fz = 0$$

$$div\ F = 2$$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$div\ F = 0? \quad \neq 0? \quad rot\ F = 0? \quad \neq 0?$$

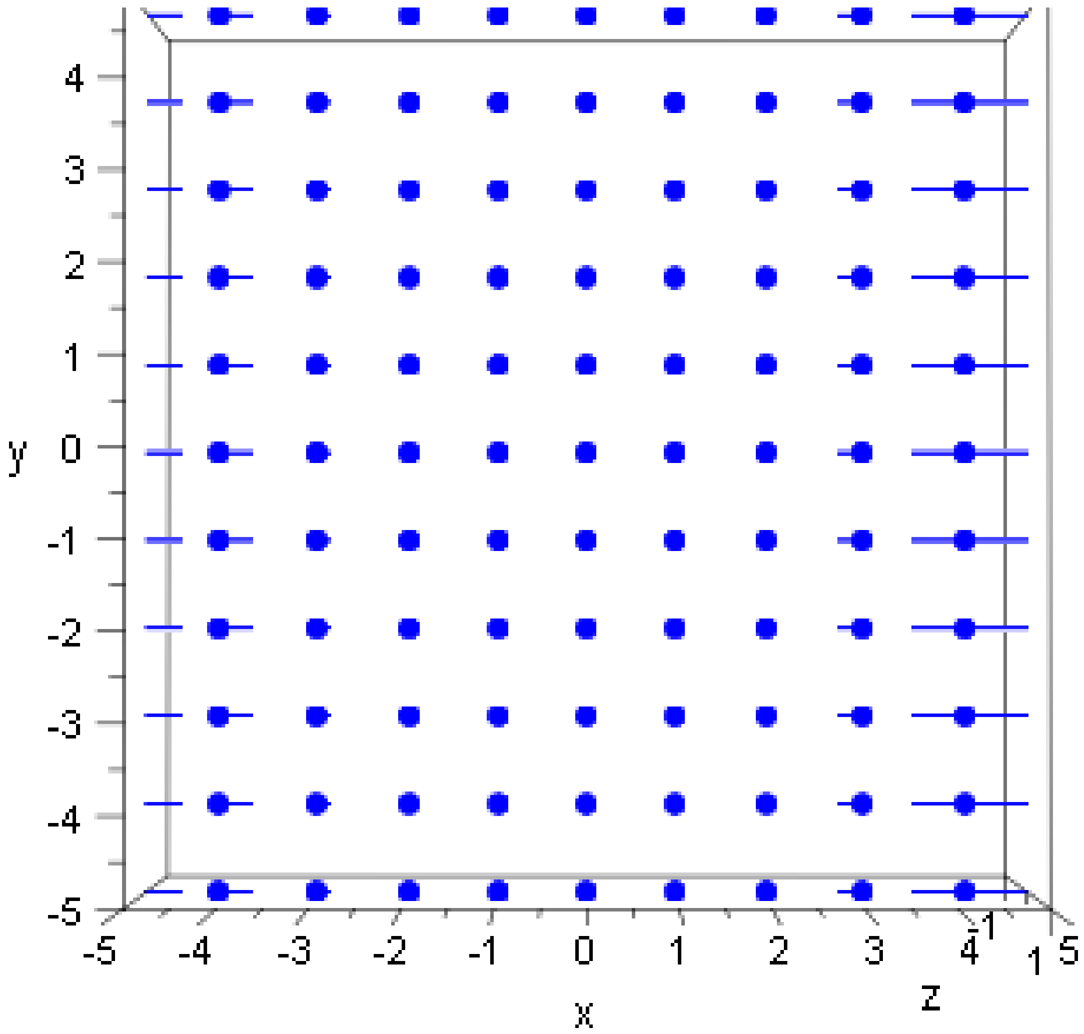


$$fx := 0$$
$$fy := x * y$$
$$fz = 0$$

$$div\ F = x$$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix}$$

$$div\ F = 0? \quad \neq 0? \quad rot\ F = 0? \quad \neq 0?$$

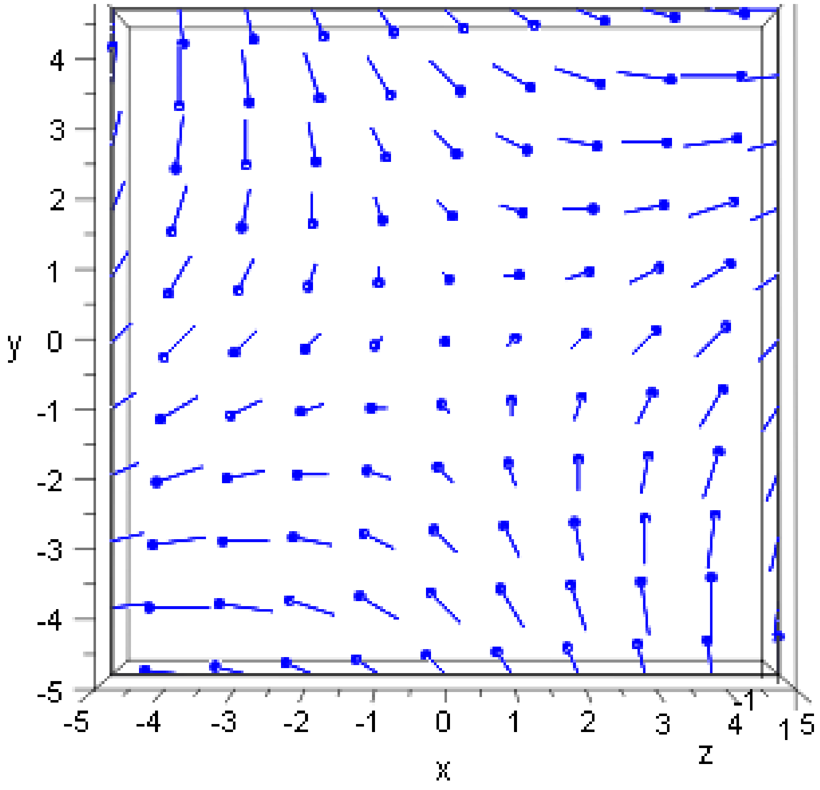


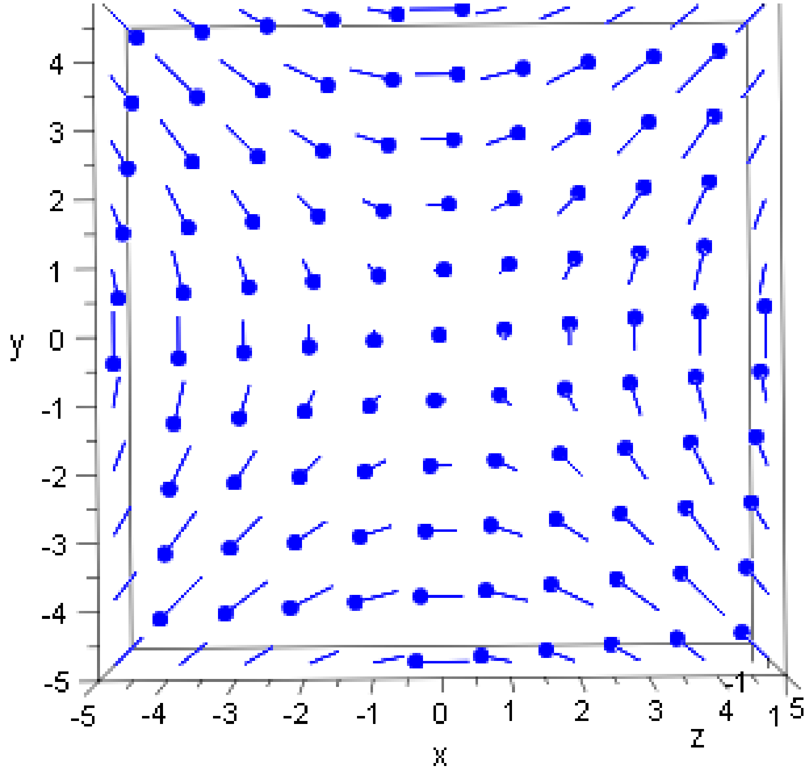
$$fx := x^3 + x^2 - 1$$
$$fy := 0$$
$$fz = 0$$

$$div\ F = 3x^2 + 2x$$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

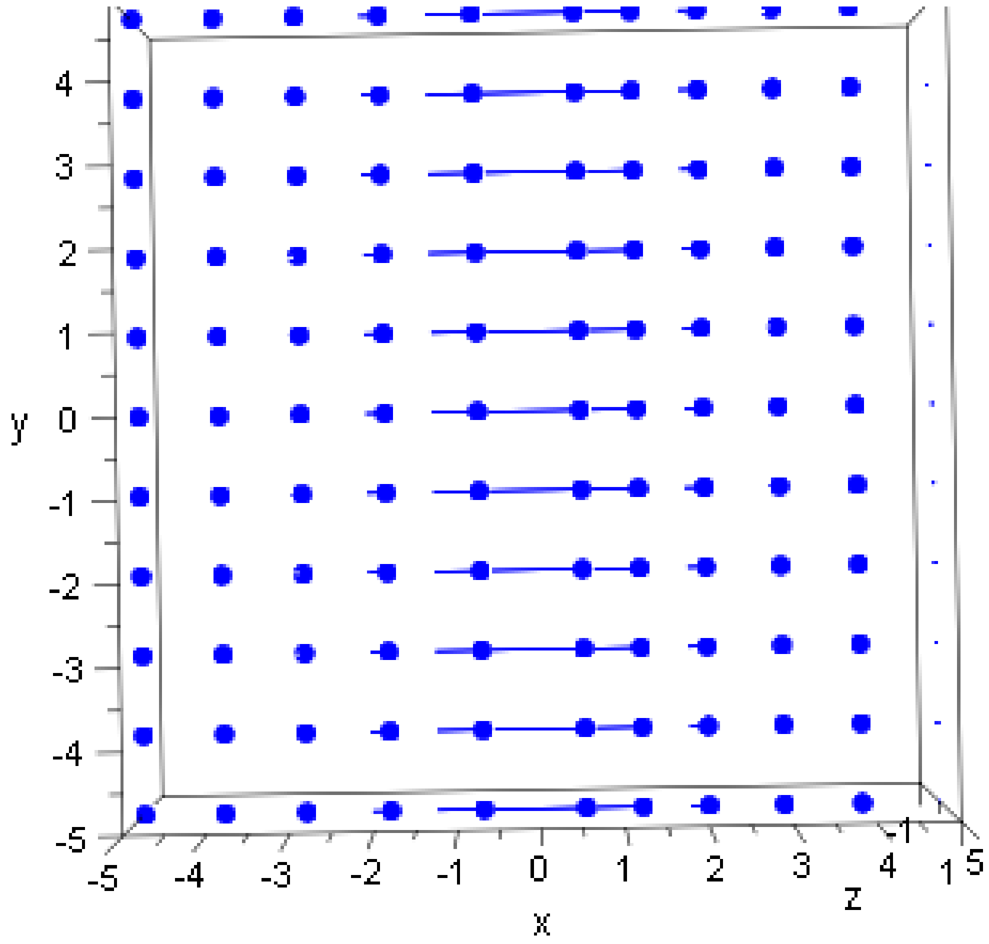
$$div\, F = 0 \qquad rot\, F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

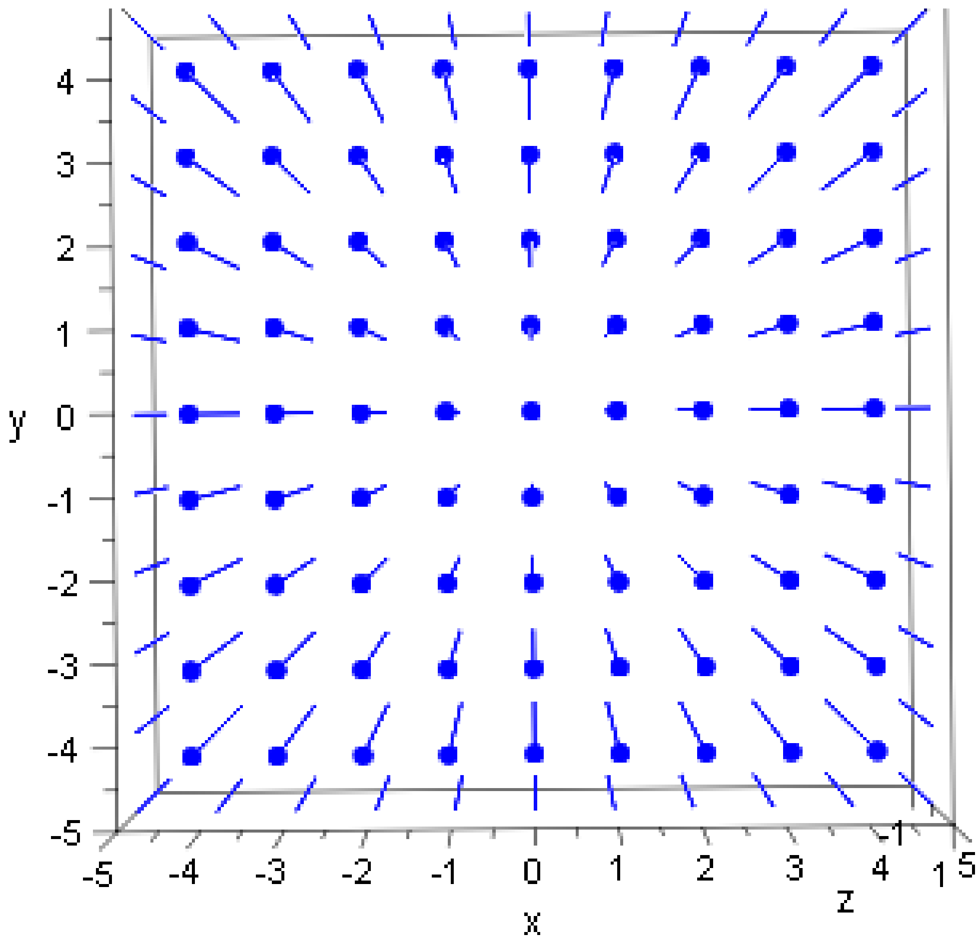




回転しないし、正味は出入りがない？

$$div\, F \neq 0 \qquad rot\, F = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

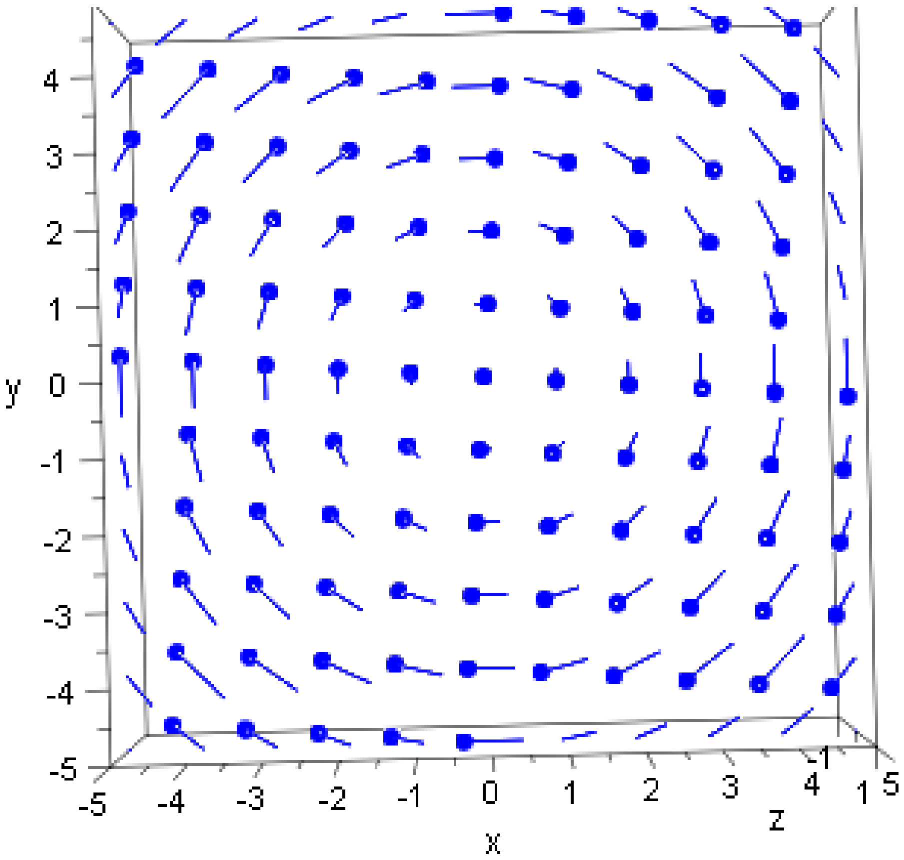




正味外に出る？

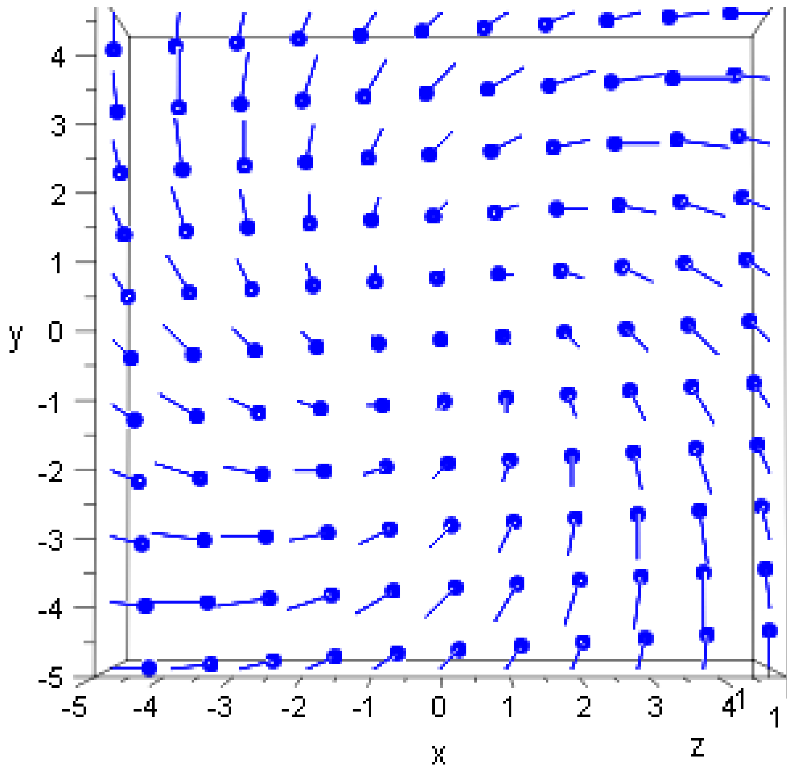
$div\ F = 0$

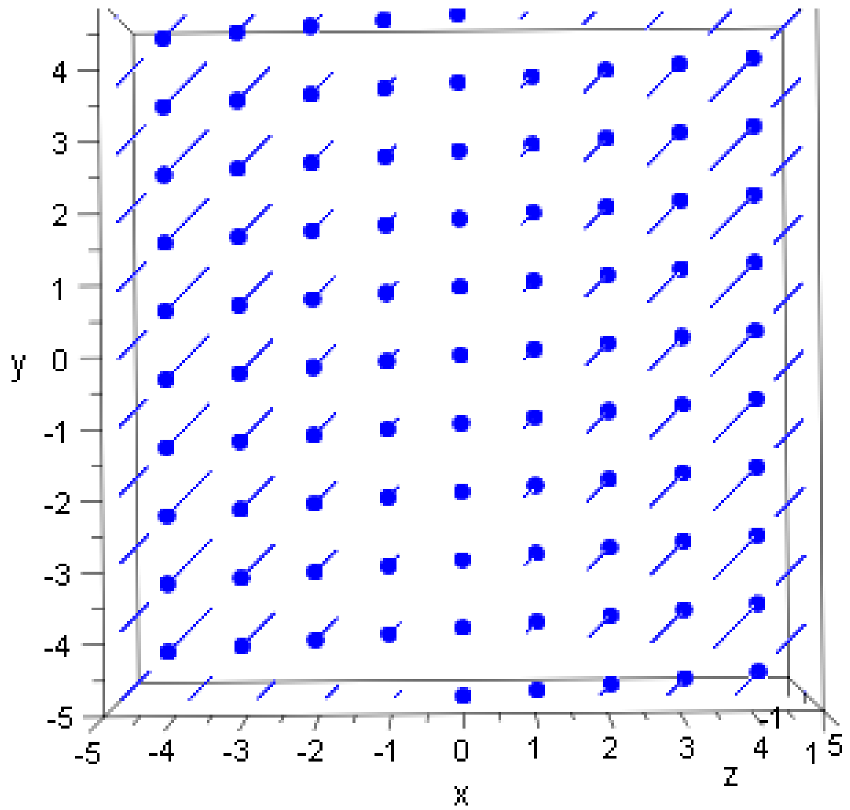
$rot\ F \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

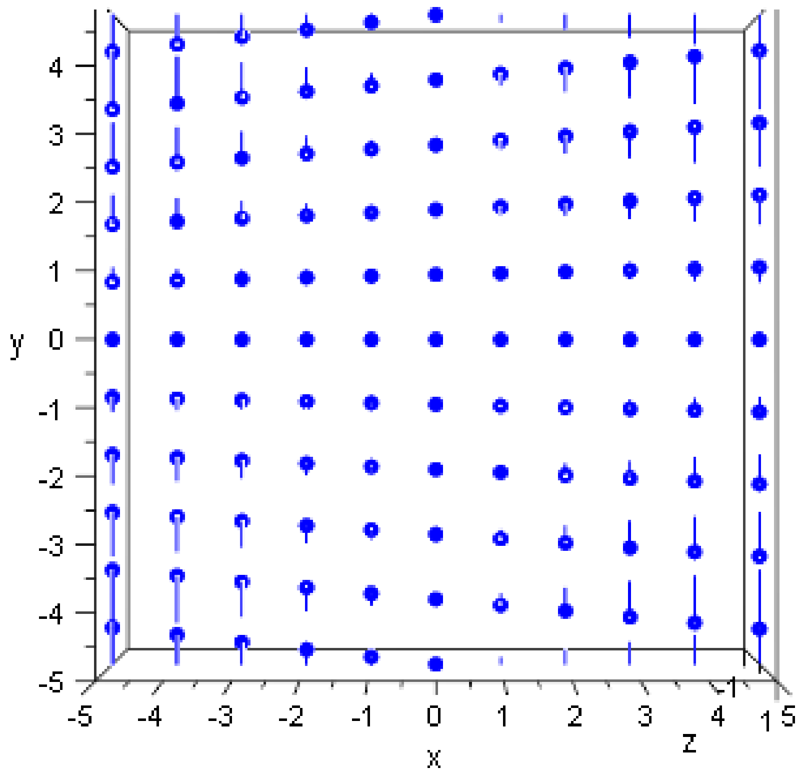


回転されてしまう？

$div\ F \neq 0$ $\quad$ $rot\ F \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$



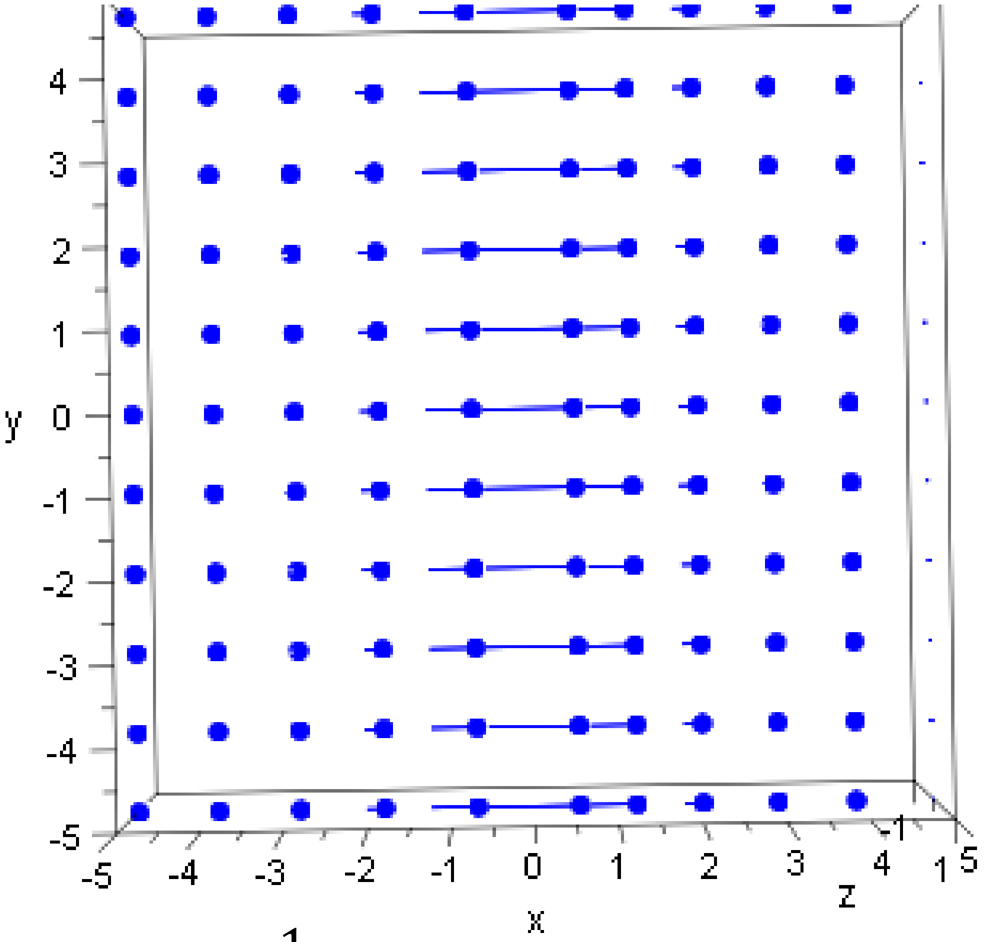


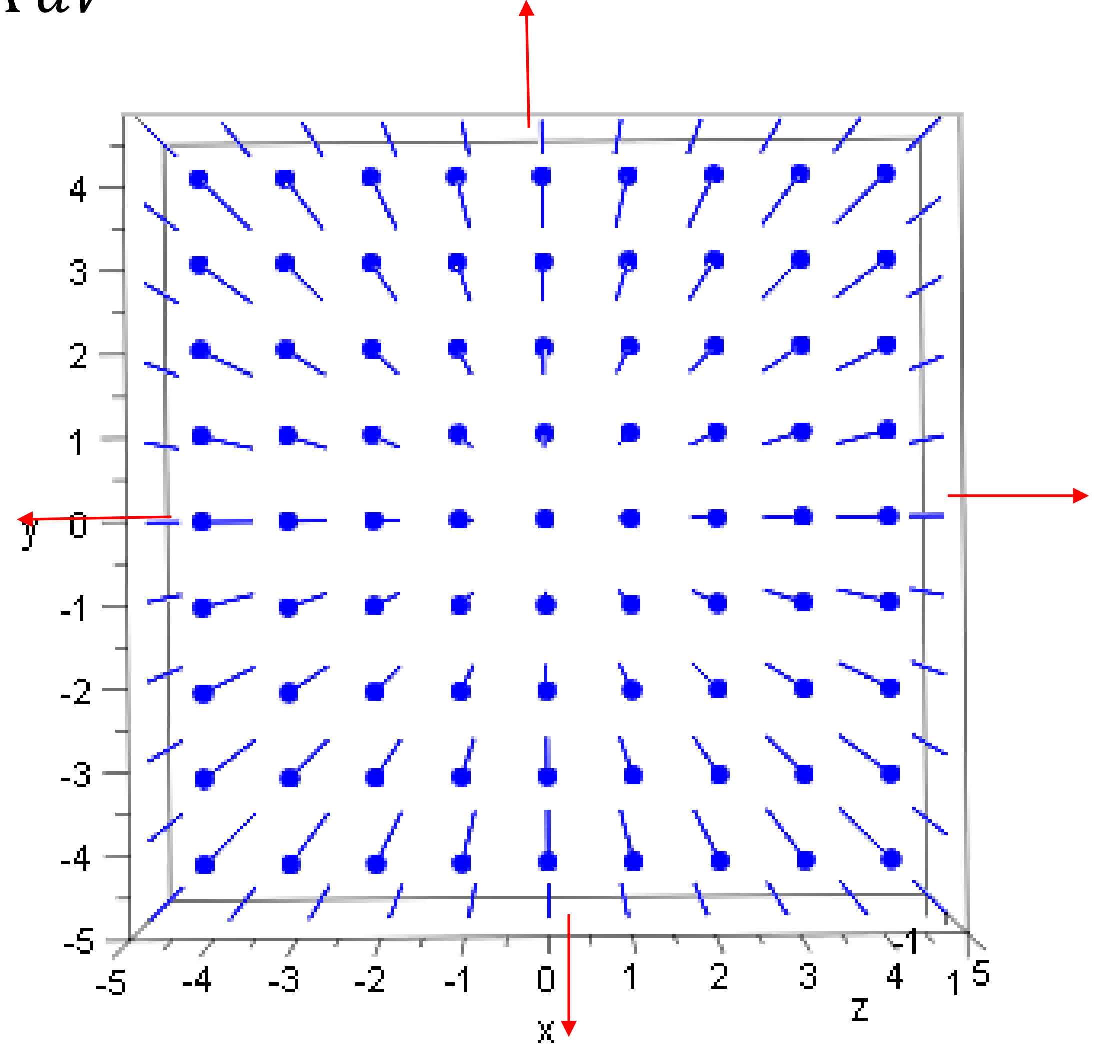


回転するし、正味の出入りがある？

$$\int_S \mathrm{A}\cdot\mathrm{n}\, dS = \int_V \nabla\cdot \mathrm{A}\, dV$$

ガウスの発散定理





$$fx = \frac{1}{1+x^2}$$

$fy = 0$

$fz = 0$

$$div\ F = \frac{-2x}{(1+x^2)^2}$$

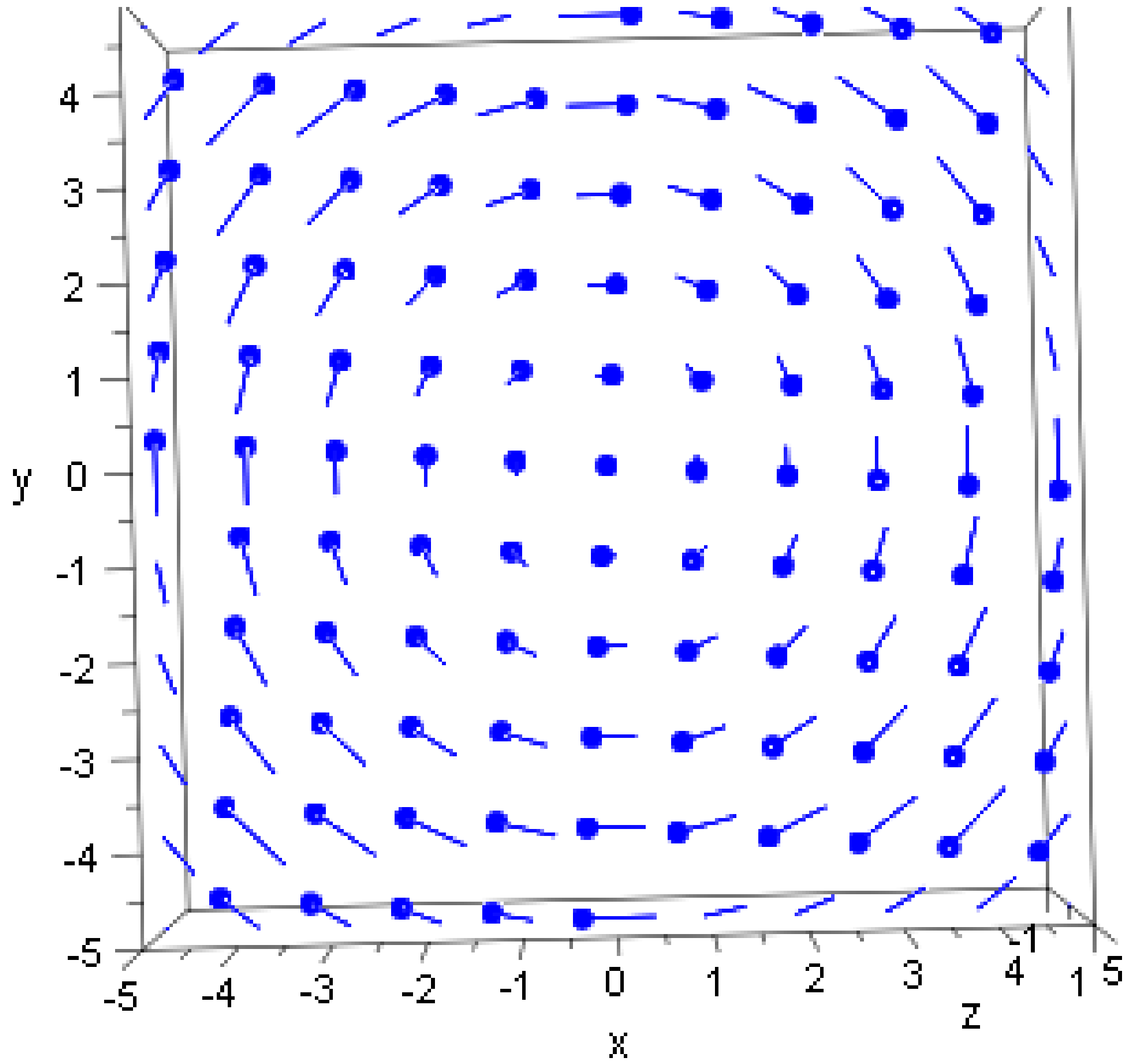
$fx := x$

$fy := y$

$fz = 0$

$$\int_C \mathrm{A} \cdot d\mathrm{l} = \int_S \nabla \times \mathrm{A} \cdot \mathrm{n}\, dS$$

ストークスの定理

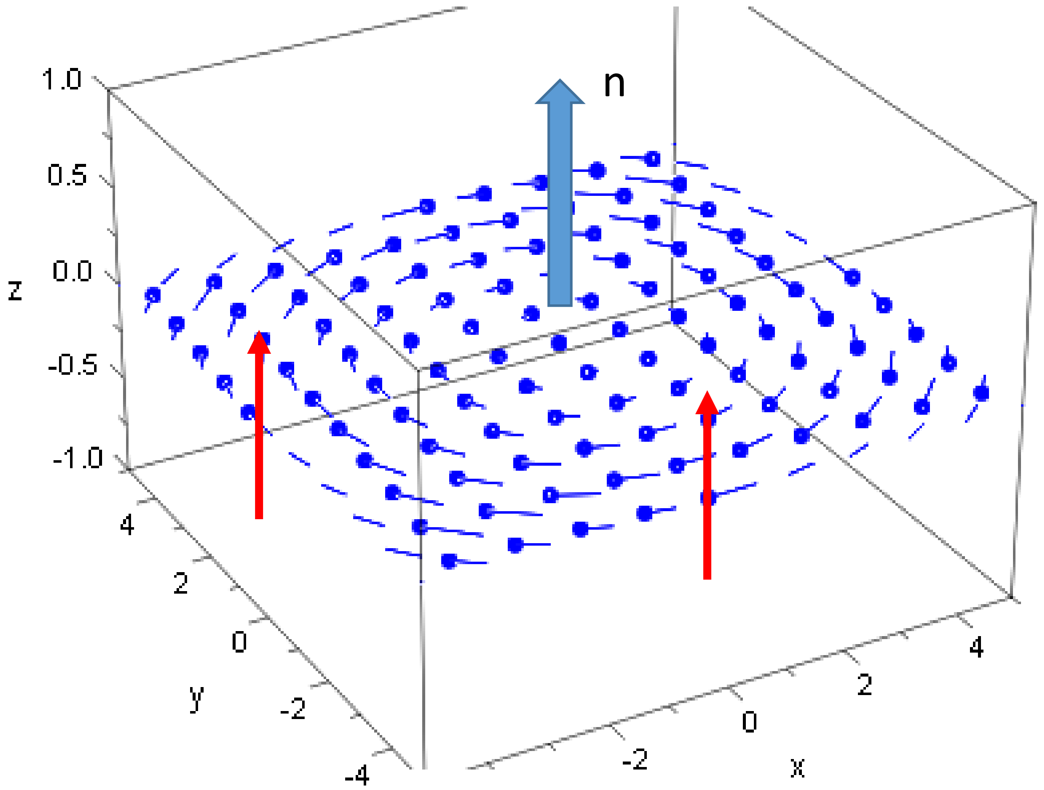


$fx := y$

$fy := -x$

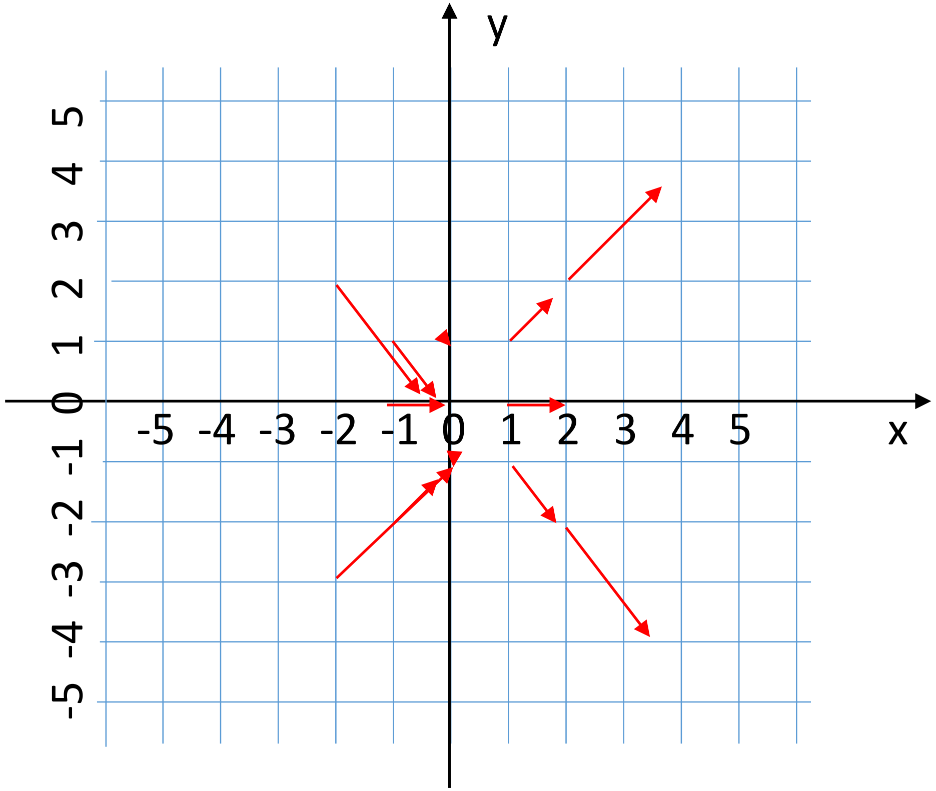
$fz = 0$

$$rot\ F = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$



確認　問題

$$\mathrm{div}\,\vec{B} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} \quad rot\,\vec{A} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad \int_S \vec{F} \cdot \vec{n} dS = \int_V div \cdot \vec{F} dV$$



$$\vec{F} = \begin{pmatrix} x^2 \\ y^2 \\ 0 \end{pmatrix} \qquad \mathrm{F} = \begin{pmatrix} x^2 \\ xy \\ 0 \end{pmatrix}$$