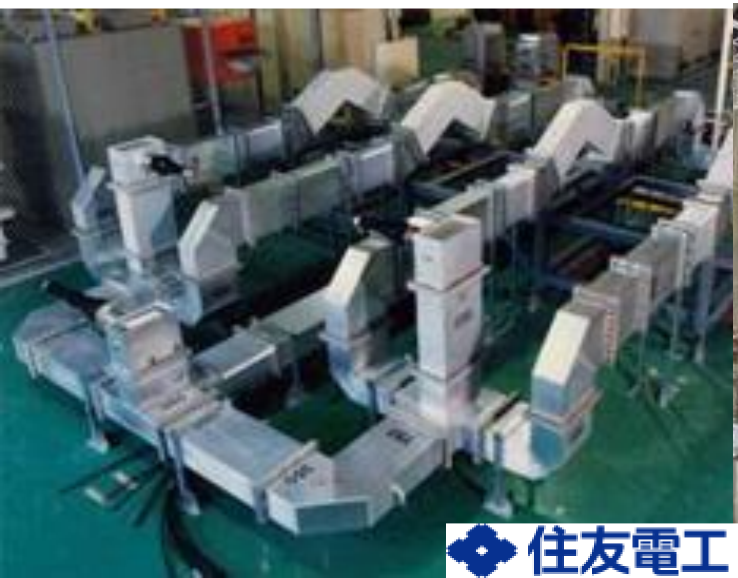
電磁波工学IX

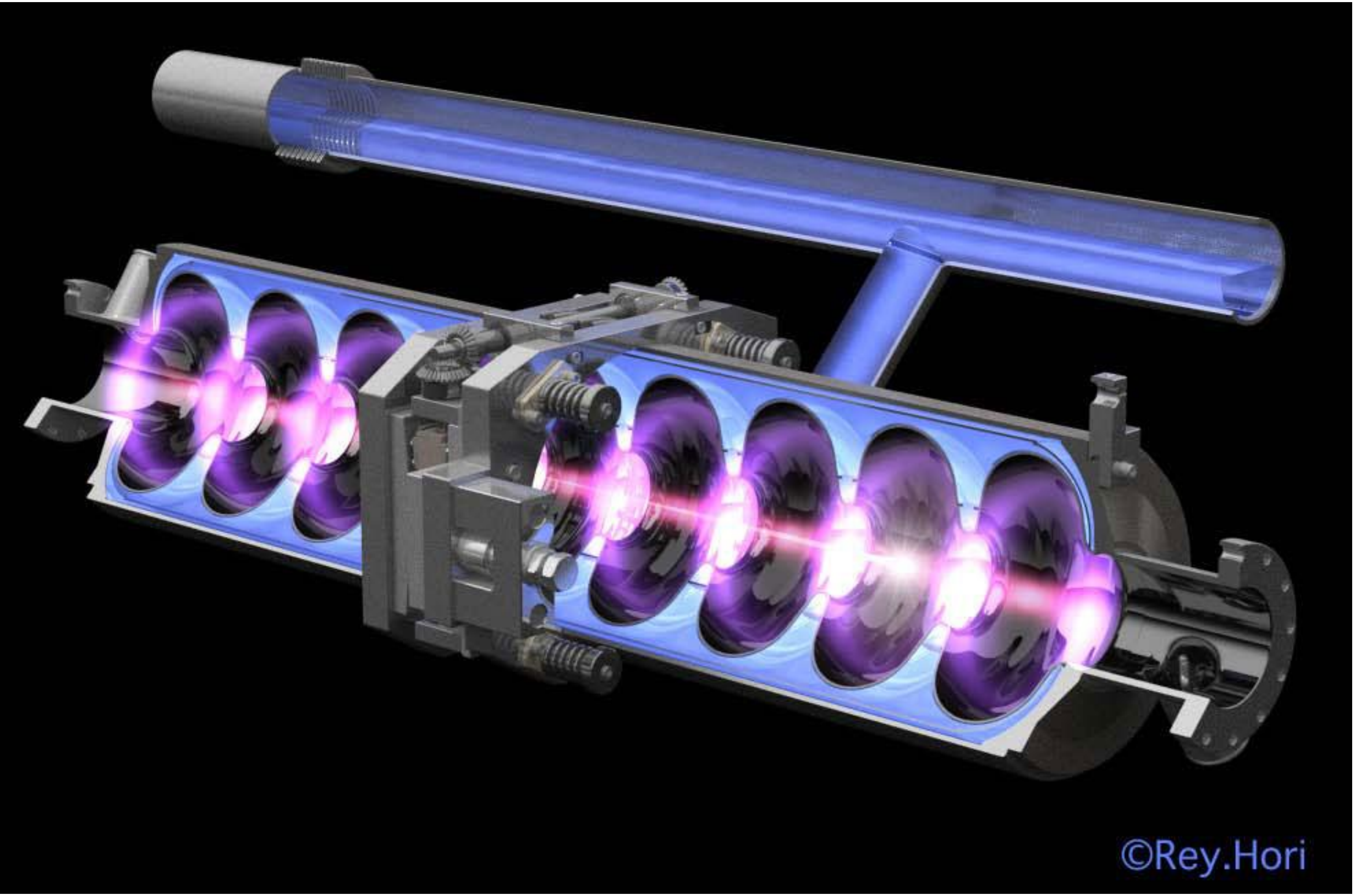
導波モードと共振器

米田仁紀




from KEK ATF

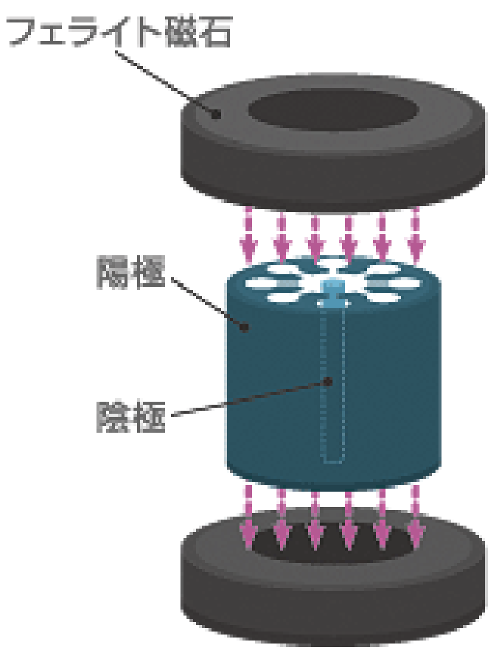

from 電気興業(株)



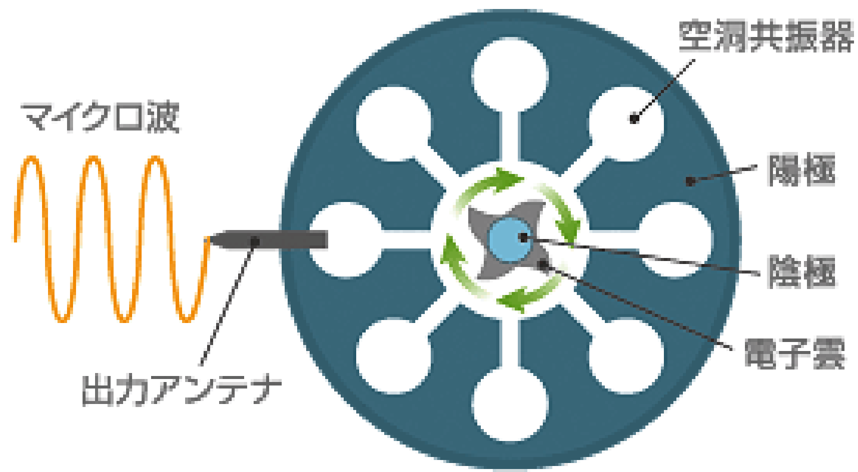
http://ilc-tsushin.kek.jp/backnumber/100315.html

もっと身近でも

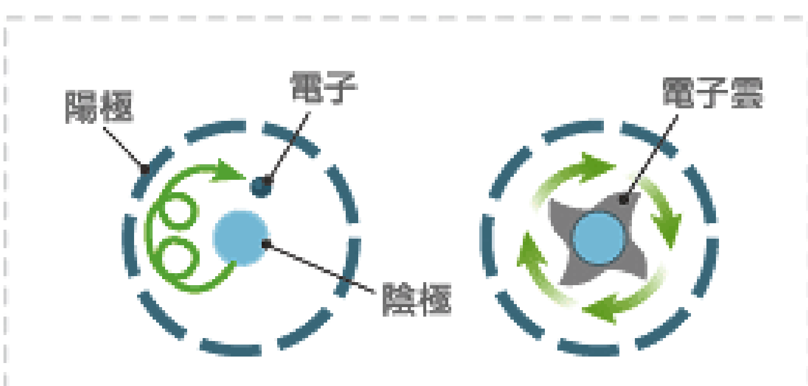
●電子レンジのマグネトロンの基本構造



●電極断面



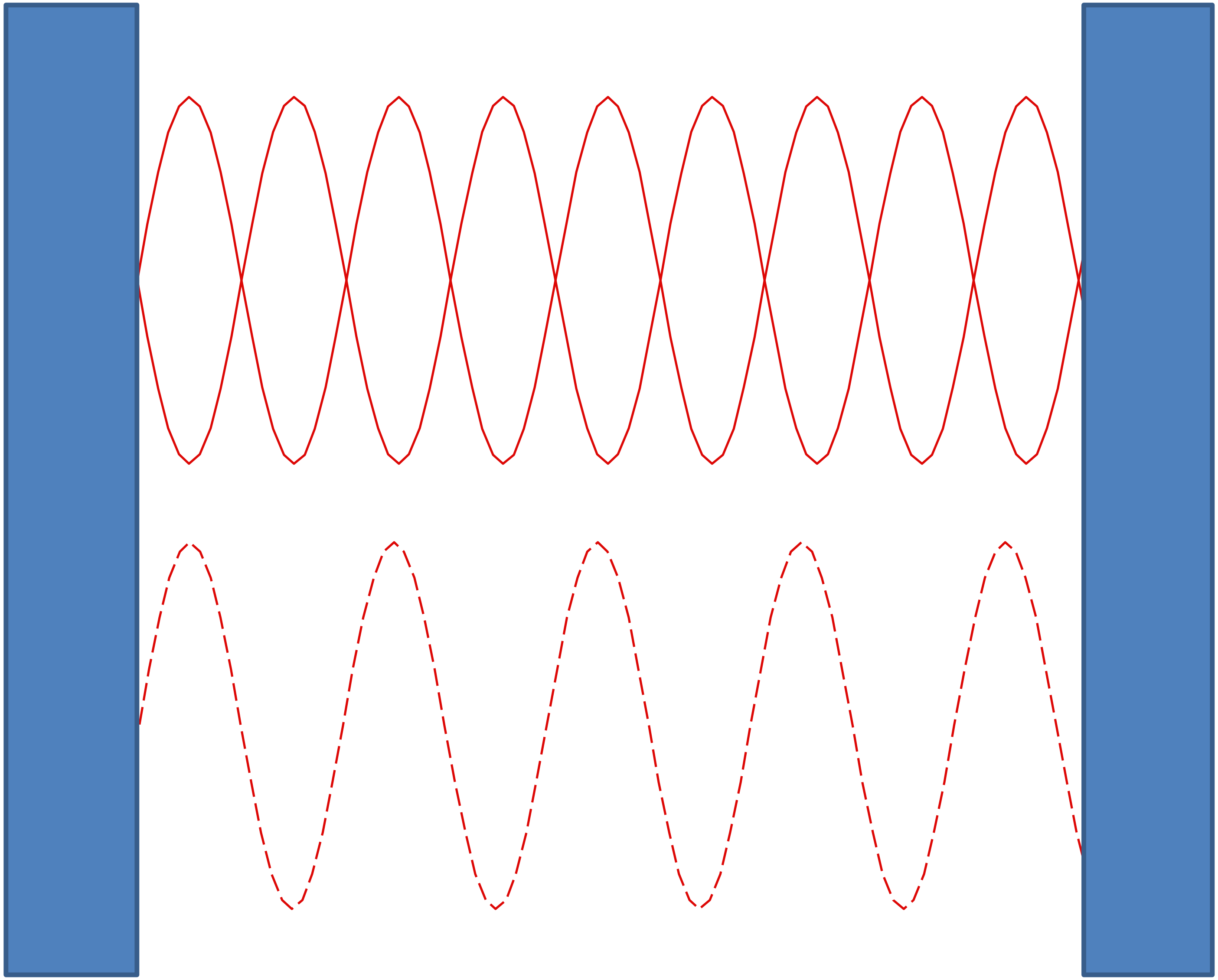
電子雲の回転は、空洞部において空洞共振を起こす。そのエネルギーをアンテナからマイクロ波として取り出す。



ローレンツ力により、電子は陰極のまわりを回転しながら周回する。その結果、歯車のような電子雲となって回転する。

https://www.tdk.co.jp/techmag/inductive/200811/index2.htm

箱の中の電磁波の閉じ込め



＝＞三次元的に

Maxwell 方程式

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\frac{\partial}{\partial t} \Rightarrow i\omega$$

$$\frac{\partial}{\partial z} \Rightarrow -ik$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -i\omega\mu_0 H_x$$

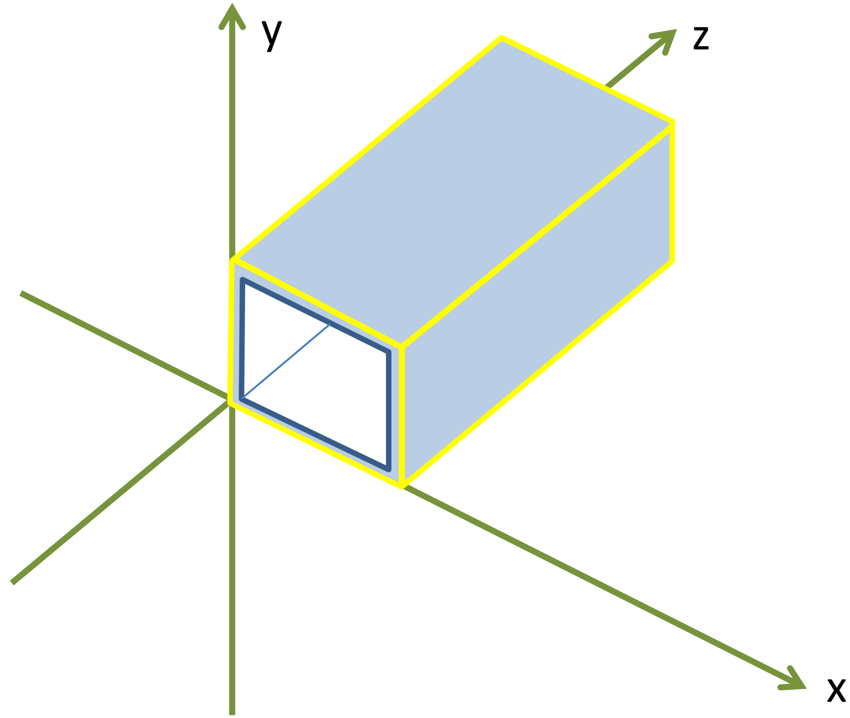
$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -i\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu_0 H_z$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = i\omega\varepsilon_0 E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i\omega\varepsilon_0 E_y$$

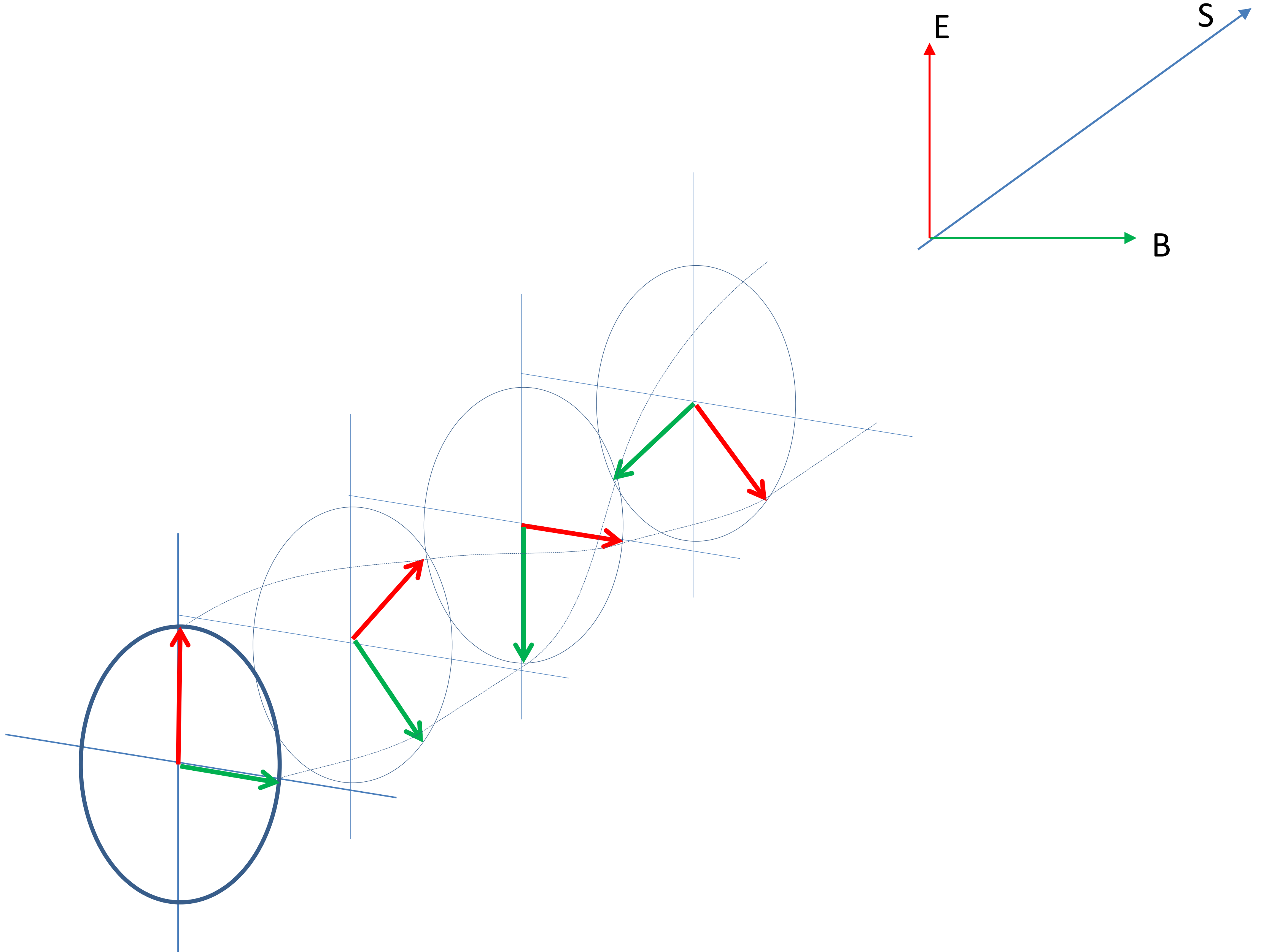
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon_0 E_z$$



1. e^{-ikz}
として電場、磁場の
z成分＝0の条件を考える。

2. 2つのケースを考える。
$E_z=0$, $H_z=0$

平面波とは違う



E_z=0

$$kE_y = -\omega\mu_0 H_x$$

$$kE_x = \omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -i\omega\mu_0 H_z$$

$$\frac{\partial H_z}{\partial y} + ikH_y = i\omega\varepsilon_0 E_x$$

$$ikH_x + \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0 E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0$$

$$\frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0 E_y + i\frac{k^2}{\omega\mu_0}E_y$$

$$= -i\frac{{k_0}^2 - k^2}{\omega\mu_0}E_y$$

$$\frac{\partial H_z}{\partial y} = i\omega\varepsilon_0 E_x - i\frac{k^2}{\omega\mu_0}E_x$$

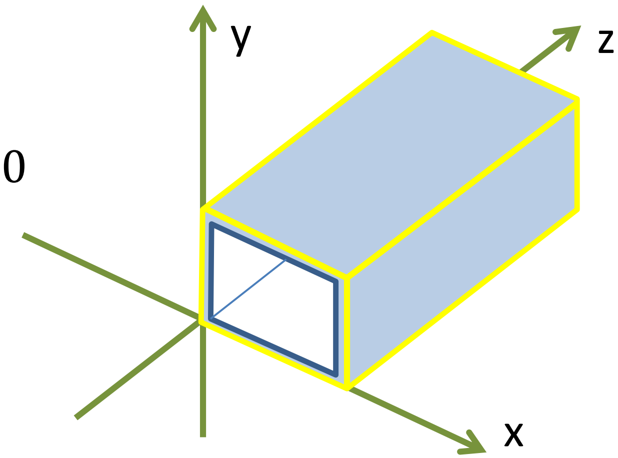
$$= i\frac{{k_0}^2 - k^2}{\omega\mu_0}E_x$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left({k_0}^2 - k^2\right)H_z = 0$$

Transverse E mode => TE mode

E_z=0

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left({k_0}^2 - k^2\right)H_z = 0$$



$$H_z = \begin{matrix}\cos\xi x \\ \sin\xi x\end{matrix}\ , \qquad \begin{matrix}\cos\eta y \\ \sin\eta y\end{matrix}$$

$$H_z = (A\cos\xi x + B\sin\xi x)(C\cos\eta y + D\sin\eta y)e^{-ikz}$$

$$(-\xi^2)H_z + (-\eta^2)H_z + \left({k_0}^2 - k^2\right)H_z = 0$$

$$(\xi^2) + (\eta^2) = \left({k_0}^2 - k^2\right)$$

p.7 より

$$\frac{\partial H_z}{\partial x} = -i\frac{{k_0}^2 - k^2}{\omega\mu_0}E_y$$

$$E_y = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial H_z}{\partial x} = i\frac{\omega\mu_0\xi}{{k_0}^2 - k^2}(-A\sin\xi x + B\cos\xi x)(C\cos\eta y + D\sin\eta y)e^{-ikz}$$

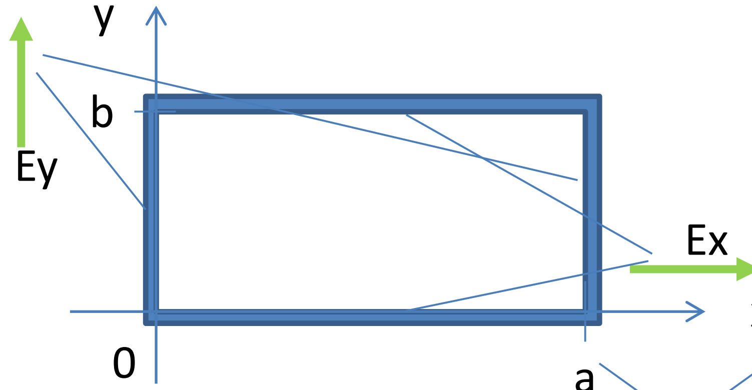
p.7 より

$$\frac{\partial H_z}{\partial y} = i\frac{{k_0}^2 - k^2}{\omega\mu_0}E_x$$

$$E_x = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial H_z}{\partial y} = -i\frac{\omega\mu_0\eta}{{k_0}^2 - k^2}(A\cos\xi x + B\sin\xi x)(-C\sin\eta y + D\cos\eta y)e^{-ikz}$$

壁が完全な金属だとすると

$$E_x = 0 \quad at\ y = 0, b$$
$$E_y = 0 \quad at\ x = 0, a$$



$$E_x = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial H_z}{\partial y} = i\frac{\omega\mu_0\eta}{{k_0}^2 - k^2}(A\cos\xi x + B\sin\xi x)(-C\sin\eta y + D\cos\eta y)e^{-ikz}$$

$$E_y = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial H_z}{\partial x} = i\frac{\omega\mu_0\xi}{{k_0}^2 - k^2}(-A\sin\xi x + B\cos\xi x)(C\cos\eta y + D\sin\eta y)e^{-ikz}$$

$\cos \quad \Rightarrow 0$ $\qquad \sin\xi a = 0, \qquad \sin\xi b = 0$

$$\xi = \frac{m\pi}{a}, \eta = \frac{n\pi}{b}$$

$$E_x = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$E_y = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

境界で決まる周期

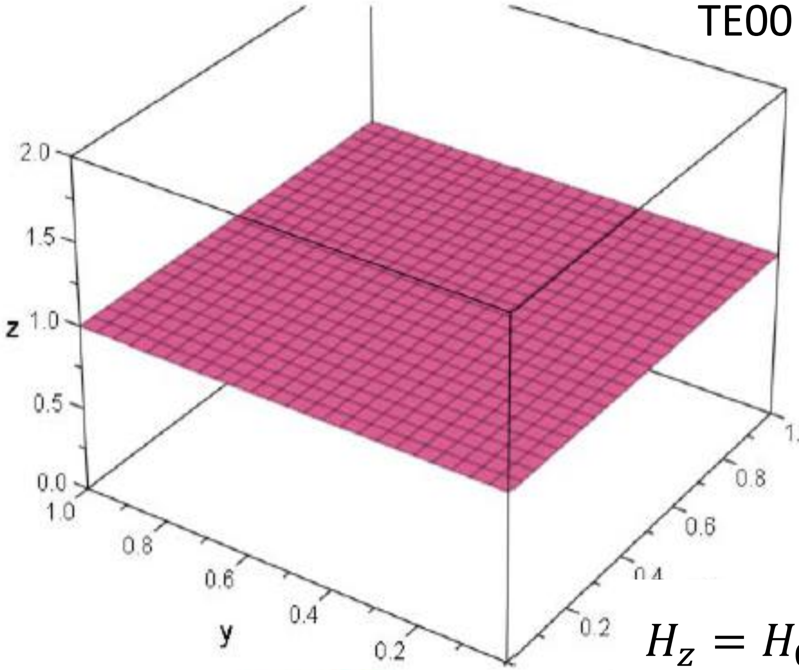
$$E_z = 0$$

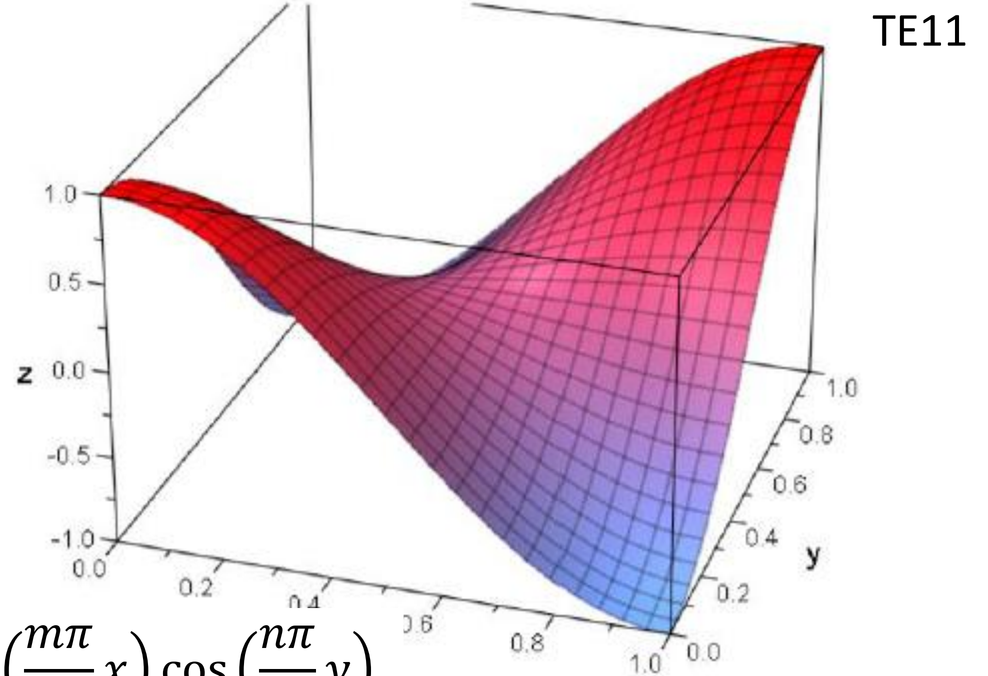
$$kE_y = -\omega\mu_0 H_x \Rightarrow H_x = i\frac{k}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$kE_x = \omega\mu_0 H_y \Rightarrow H_y = i\frac{k}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

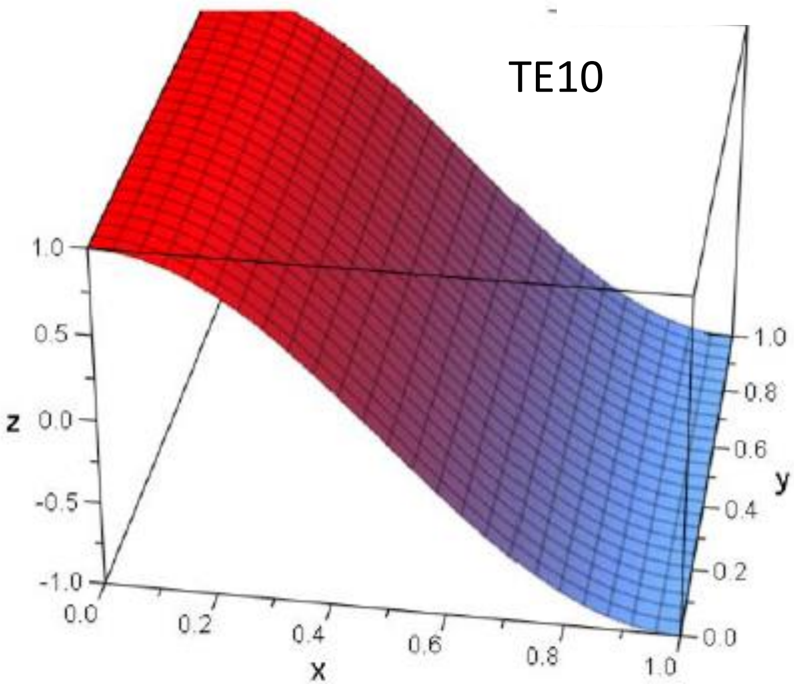
$$\frac{\partial H_z}{\partial y} = i\frac{{k_0}^2 - k^2}{\omega\mu_0}E_x$$

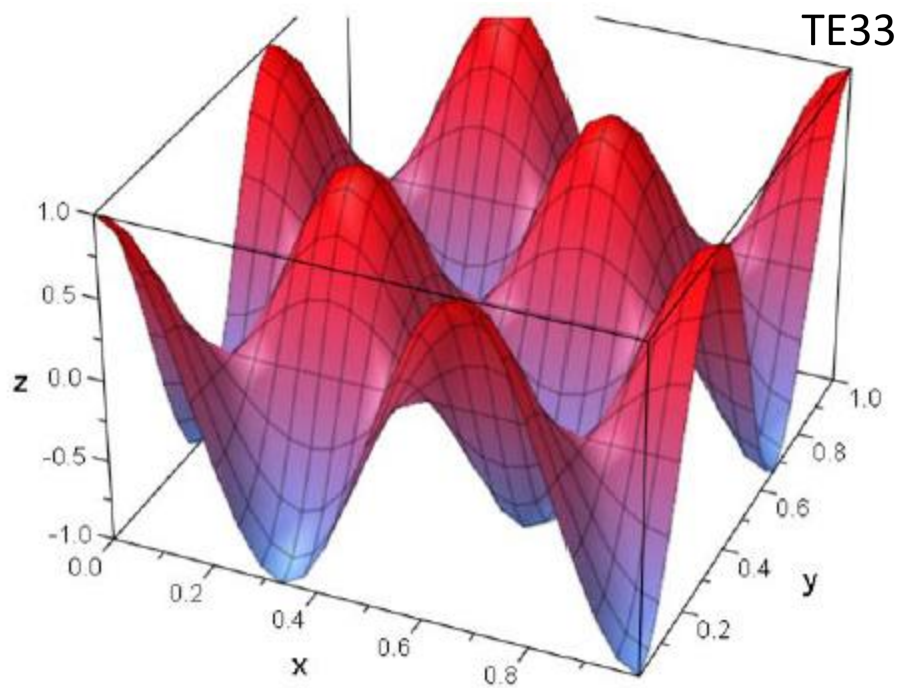
$$H_z = H_0\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$





$$H_z = H_0 \cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)$$





H_z=0

$$kH_y = \omega\varepsilon_0 E_x$$

$$kH_x = -\omega\varepsilon_0 E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon_0 E_z$$

$$\frac{\partial E_z}{\partial y} + ikE_y = -i\omega\mu_0 H_x$$

$$ikE_x + \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0$$

$$\frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y - i\frac{k^2}{\omega\varepsilon_0}H_y = i\frac{{k_0}^2 - k^2}{\omega\varepsilon_0}H_y$$

$$\frac{\partial E_z}{\partial y} = -i\omega\mu_0 H_x + i\frac{k^2}{\omega\varepsilon_0}H_x = -i\frac{{k_0}^2 - k^2}{\omega\varepsilon_0}H_x$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \left({k_0}^2 - k^2\right)E_z = 0$$

Transverse H mode => TM mode

E_z=0

$$\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+\left({k_0}^2-k^2\right)H_z=0$$

$H_z =$
$(A\cos\xi x+B\sin\xi x)(C\cos\eta y+D\sin\eta y)e^{-ikz}$

H_z=0

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\left({k_0}^2-k^2\right)E_z=0$$

$E_z =$
$(A\cos\xi x+B\sin\xi x)(C\cos\eta y+D\sin\eta y)e^{-ikz}$

$$(-\xi^2)E_z+(-\eta^2)E_z+\left({k_0}^2-k^2\right)E_z=0$$

$$(\xi^2)+(\eta^2)=\left({k_0}^2-k^2\right)$$

$$\frac{\partial E_z}{\partial x}=i\frac{{k_0}^2-k^2}{\omega\mu_0}H_y$$

$$H_y=-i\frac{\omega\mu_0}{{k_0}^2-k^2}\frac{\partial E_z}{\partial x}=-i\frac{\omega\mu_0\xi}{{k_0}^2-k^2}(-A\sin\xi x+B\cos\xi x)(C\cos\eta y+D\sin\eta y)e^{-ikz}$$

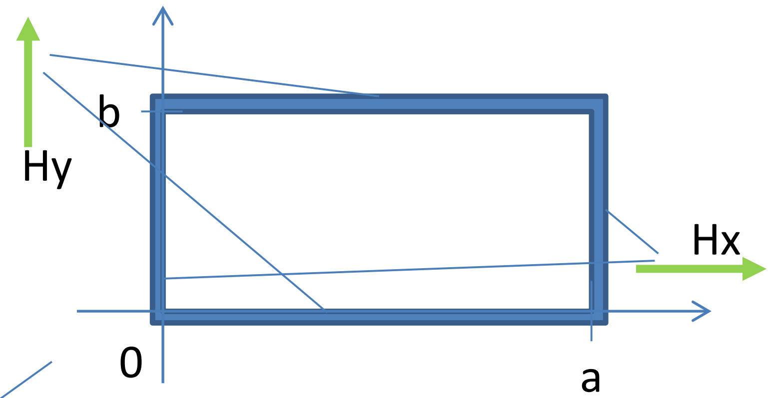
$$\frac{\partial E_z}{\partial y}=-i\frac{{k_0}^2-k^2}{\omega\mu_0}H_x$$

$$H_x=i\frac{\omega\mu_0}{{k_0}^2-k^2}\frac{\partial E_z}{\partial y}=i\frac{\omega\mu_0\eta}{{k_0}^2-k^2}(A\cos\xi x+B\sin\xi x)(-C\sin\eta y+D\cos\eta y)e^{-ikz}$$

壁が金属

$E_x = 0 \quad at\ y = 0, b$
$E_y = 0 \quad at\ x = 0, a$

$H_x = 0 \quad at\ x = 0, a$
$H_y = 0 \quad at\ y = 0, b$



$$H_x = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial E_z}{\partial y} = i\frac{\omega\mu_0\eta}{{k_0}^2 - k^2}(A\cos\xi x + B\sin\xi x)(-C\sin\eta y + D\cos\eta y)e^{-ikz}$$

$$H_y = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\frac{\partial E_z}{\partial x} = -i\frac{\omega\mu_0\xi}{{k_0}^2 - k^2}(-A\sin\xi x + B\cos\xi x)(C\cos\eta y + D\sin\eta y)e^{-ikz}$$

$\cos \Rightarrow 0$ $\qquad$ $\sin\xi a = 0, \quad \sin\xi b = 0$

$$\xi = \frac{m\pi}{a}, \eta = \frac{n\pi}{b}$$

$$H_x = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)E_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$H_y = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)E_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$H_z = 0$$

$$kH_y = -\omega\mu_0 E_x \quad \Rightarrow \quad E_x = i\frac{k}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)E_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$kH_x = \omega\mu_0 E_y \quad \Rightarrow \quad E_y = i\frac{k}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)E_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$\frac{\partial E_z}{\partial y} = i\frac{{k_0}^2 - k^2}{\omega\mu_0}H_x \qquad E_z = E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

特徴的な周波数依存性（遮断周波数）

E_z=0でのcut off

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \left({k_0}^2 - k^2\right)H_z = 0$$

$$(-\xi^2)H_z + (-\eta^2)H_z + \left({k_0}^2 - k^2\right)H_z = 0$$

$$(\xi^2) + (\eta^2) = \left({k_0}^2 - k^2\right)$$

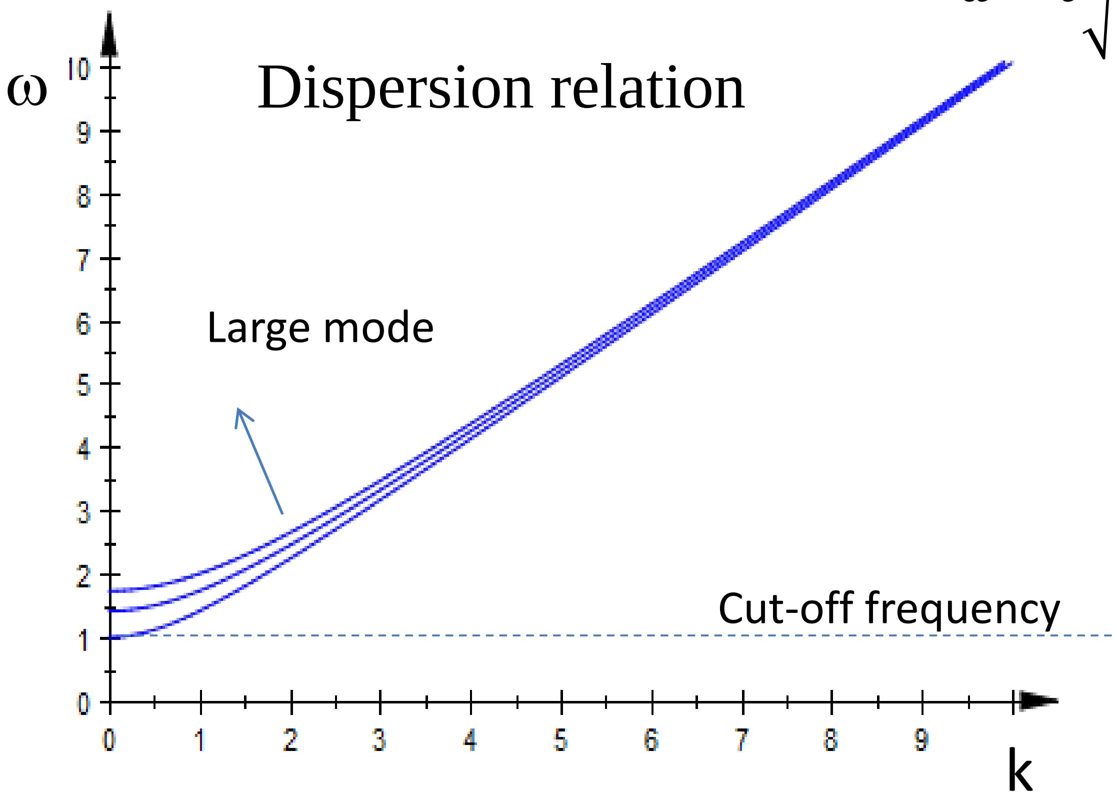
$$k = \sqrt{{k_0}^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \qquad k_0 = \frac{\omega}{c}$$

分散関係でみると

$$(\xi^2) + (\eta^2) = \left(k_0{}^2 - k^2\right)$$

$$\xi = \frac{m\pi}{a}, \eta = \frac{n\pi}{b}$$

$$\omega = c\sqrt{k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$



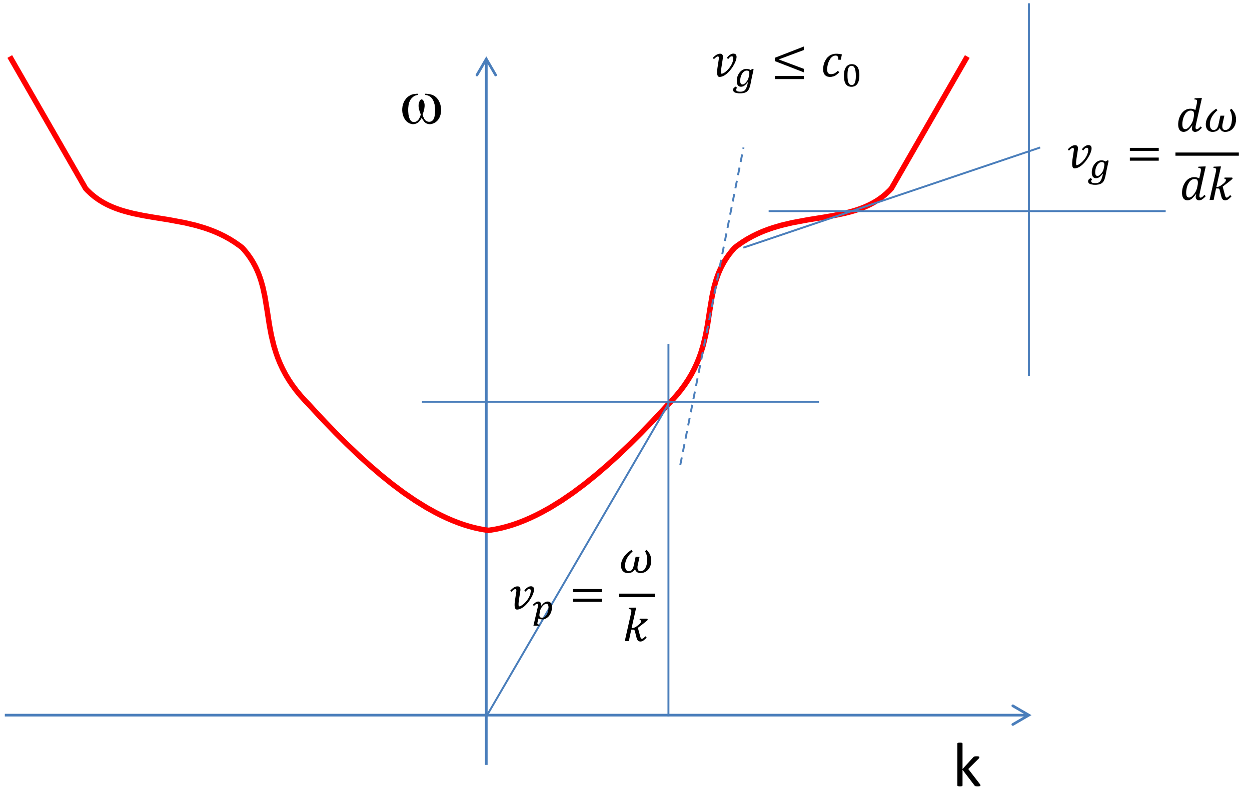
$$f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\lambda_c = \frac{2\pi}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

ある波長以下しか通らない

Dispersion relationとは？

群速度　位相速度　分散関係

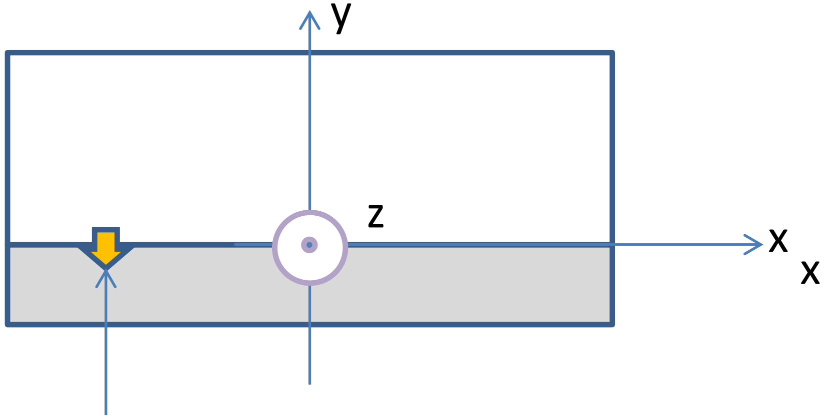


損失を考える

$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{j} = \sigma \mathbf{E}$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\sigma\mu \frac{\partial \mathbf{E}}{\partial t} - \varepsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2}$$



電磁場の浸み込みによる損失

$$\left(k_x{}^2 + k_y{}^2\right) E_z = i\omega\sigma\mu E_z + \omega^2 \varepsilon\mu E_z$$

$$k_y{}^2 = i\omega\sigma\mu + \omega^2 \varepsilon\mu$$

通常金属では第1項>>第2項（誘電性より導電率）

$$k_y = \pm\sqrt{\frac{\omega\sigma\mu}{2}}(1+i) = \pm\frac{1+i}{\delta}$$

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$$

$$-\frac{\partial H_x}{\partial y} = E_0 e^{y/\delta} e^{-i(\frac{y}{\delta}+\omega t)}$$

$$H_x = H_0 e^{y/\delta} e^{-i(\frac{y}{\delta}+\omega t)}$$
$$E_y = \frac{H_0(i-1)}{\sigma\delta} e^{y/\delta} e^{-i(\frac{y}{\delta}+\omega t)}$$

$$\langle P \rangle = \frac{1}{2} Re[E_z H_x] = -\frac{H_0 H*_0}{2\sigma\delta}$$

損失の見積り

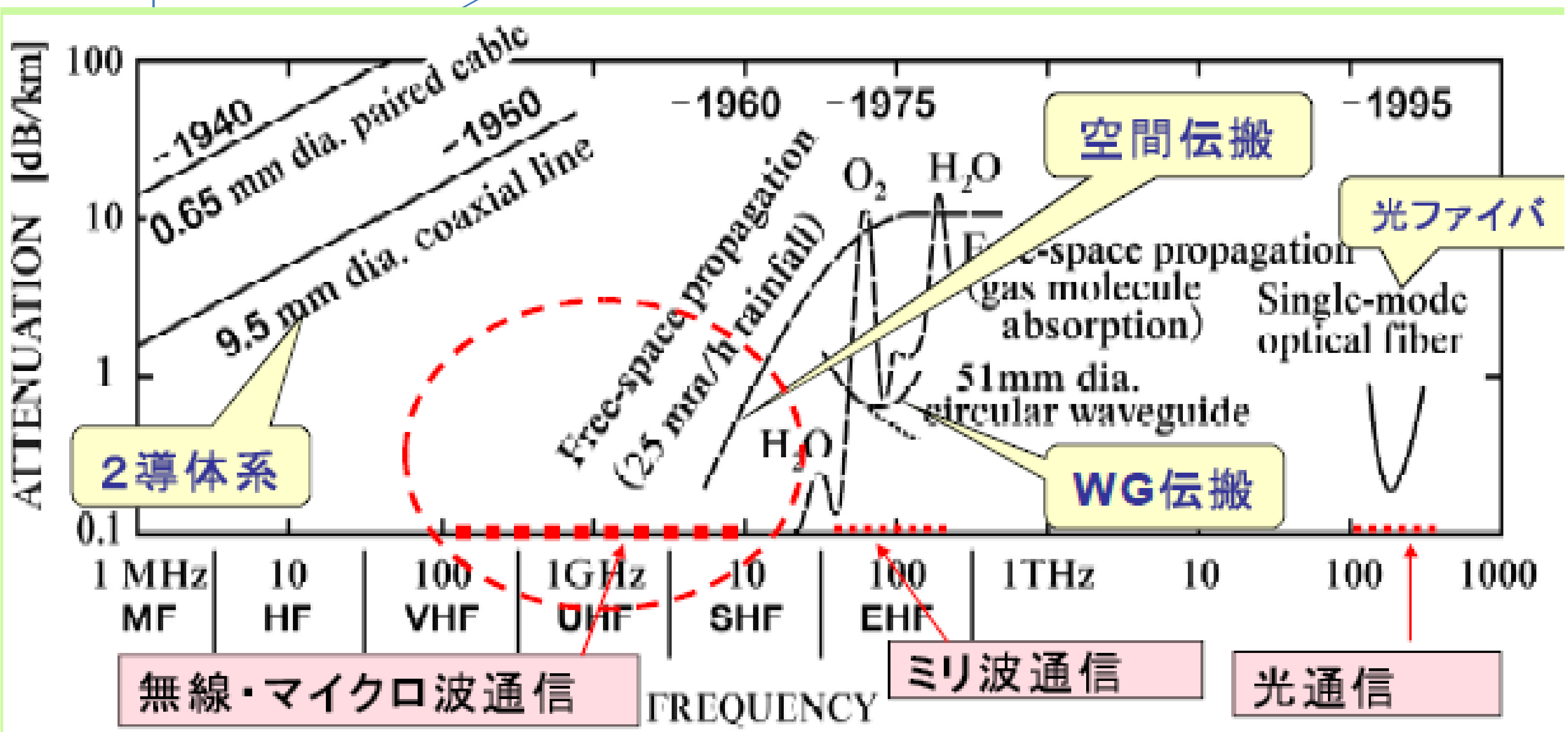
$$\frac{d\langle P\rangle}{dz} = \frac{1}{2\sigma\delta}\oint HH^* dl = \frac{{H_0}^2}{\sigma\delta}\left[\left(\frac{k_z}{k_c}\right)^2 \frac{a}{2} + \frac{a}{2} + b\right]$$

$$k_c = \frac{\pi}{a}$$

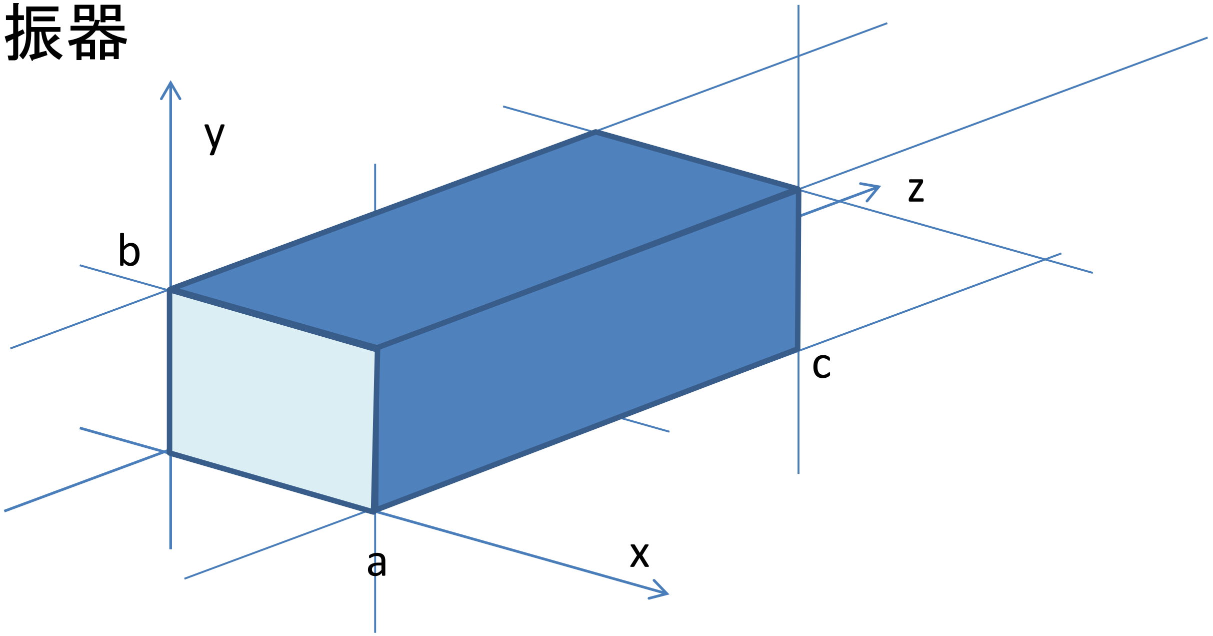
$${k_z}^2 = {k_0}^2 - {k_c}^2$$

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$$

Attenuation



共振器



$$E_x = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$\Rightarrow i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\left(Ae^{-ikz} + Be^{ikz}\right)$$

$$E_y = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\left(Ae^{-ikz} + Be^{ikz}\right)$$

z=0, c で電場が0となる

$$A + B = 0$$
$$Ae^{-ikc} + Be^{ikc} = 0$$

$$e^{-ikc} - e^{ikc} = i2\sin kc = 0$$

$$k = l\frac{\pi}{c} \quad l = 1,2,3,\ldots$$

共振器(cont.)

$$(\xi^2) + (\eta^2) = \left(k_0{}^2 - k^2\right)$$

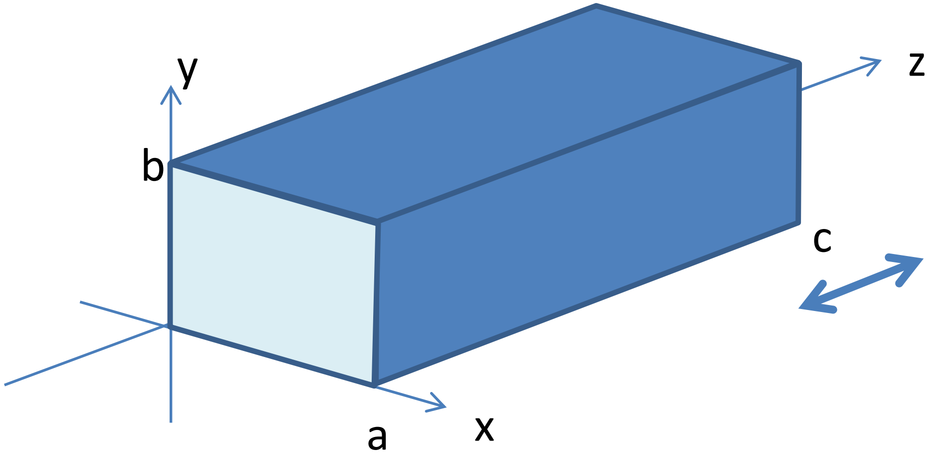
⇒

$$(\xi^2) + (\eta^2) + k^2 = \left(k_0{}^2\right)$$

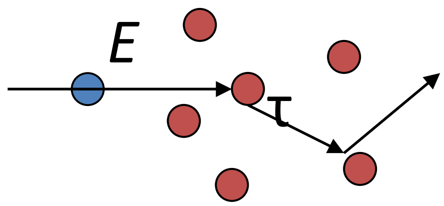
$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2 = k_0{}^2$$

この時の周波数は

$$\omega_r = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$



導電率　＝＞電子の運動で考える



$$m\left(\frac{d}{dt}+\frac{1}{\tau}\right)\delta v(t)=F=-eE\exp[-j\omega t]$$

$\delta v(t)=\delta v_0\exp[-j\omega t]$　とすると　$\delta v_0=-\dfrac{e\tau/m}{1-j\omega\tau}E$

$J=nq\delta v=\dfrac{ne^2\tau}{m(1-j\omega\tau)}E$　であるから

電気伝導度σはしたがって

$$\sigma(\omega)=\frac{ne^2\tau}{m(1-j\omega\tau)}=\frac{ne^2\tau}{m}\frac{1+j\omega\tau}{1+(\omega\tau)^2}$$

この解析でω->0とすると直流の導電率

$$\sigma_{DC}=\frac{ne^2\tau}{m}$$

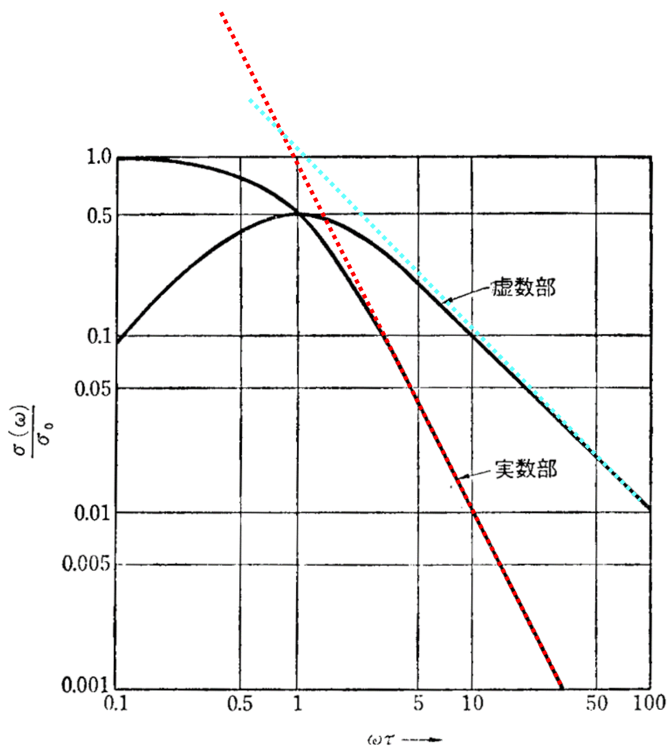


図 8・1　伝導率と ωτ との両対数関係．ω は角振動数，τ は緩和時間，

C.Kittel固体物理学入門

実際の金属の周波数依存導電率

$$\sigma = \frac{Ne^2}{m(g - i\omega)}$$

$$\hat{\varepsilon} = \hat{n}^2 = 1 + \frac{i\sigma}{\varepsilon_0 \omega} = 1 - \frac{Ne^2}{m\varepsilon_0}\frac{1}{\omega(\omega + ig)}$$

$$= 1 - \frac{\omega_c^2}{(\omega^2 + g^2)} + i\frac{\omega_c^2}{(\omega^2 + g^2)}\frac{g}{\omega}$$

$$\hat{n} = n + ik$$

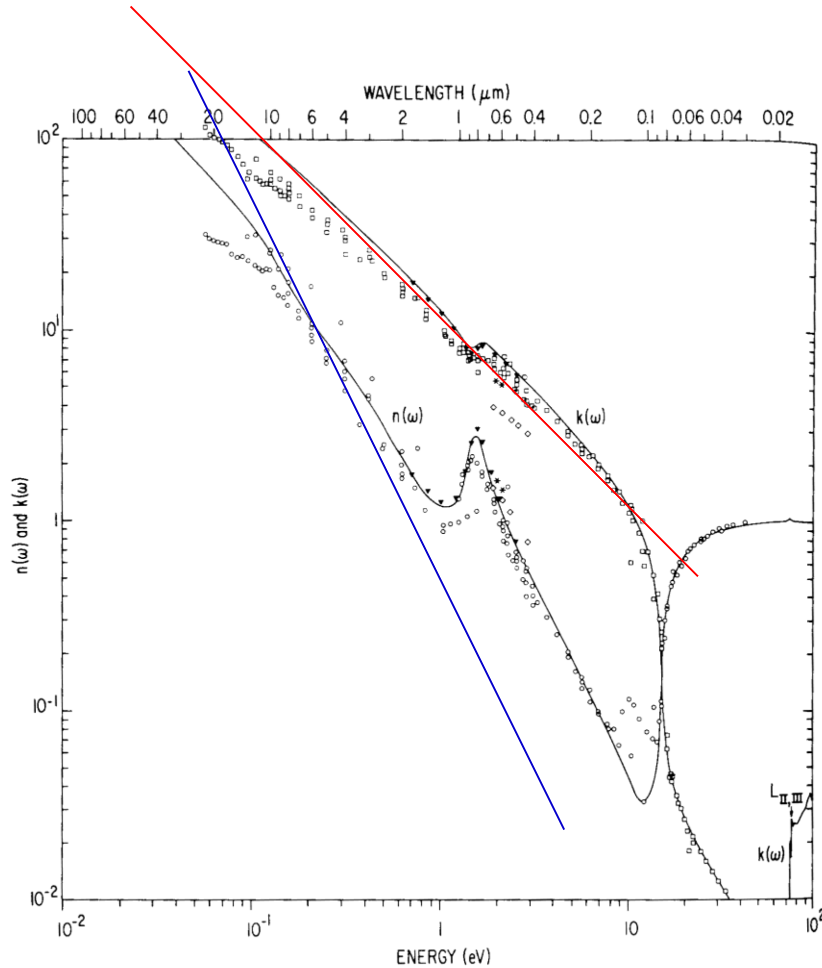
$$\mathrm{Re}\cdot\hat{\varepsilon}(\omega) \equiv n^2 - k^2$$

$$\mathrm{Im}\cdot\hat{\varepsilon}(\omega) \equiv 2nk$$

$$\sigma_{DC} = \frac{Ne^2}{mg} \sim 3\times 10^7[\Omega^{-1}m]$$

n_e for Al ~$6\text{x}10^{22}\text{cm}^{-3}$

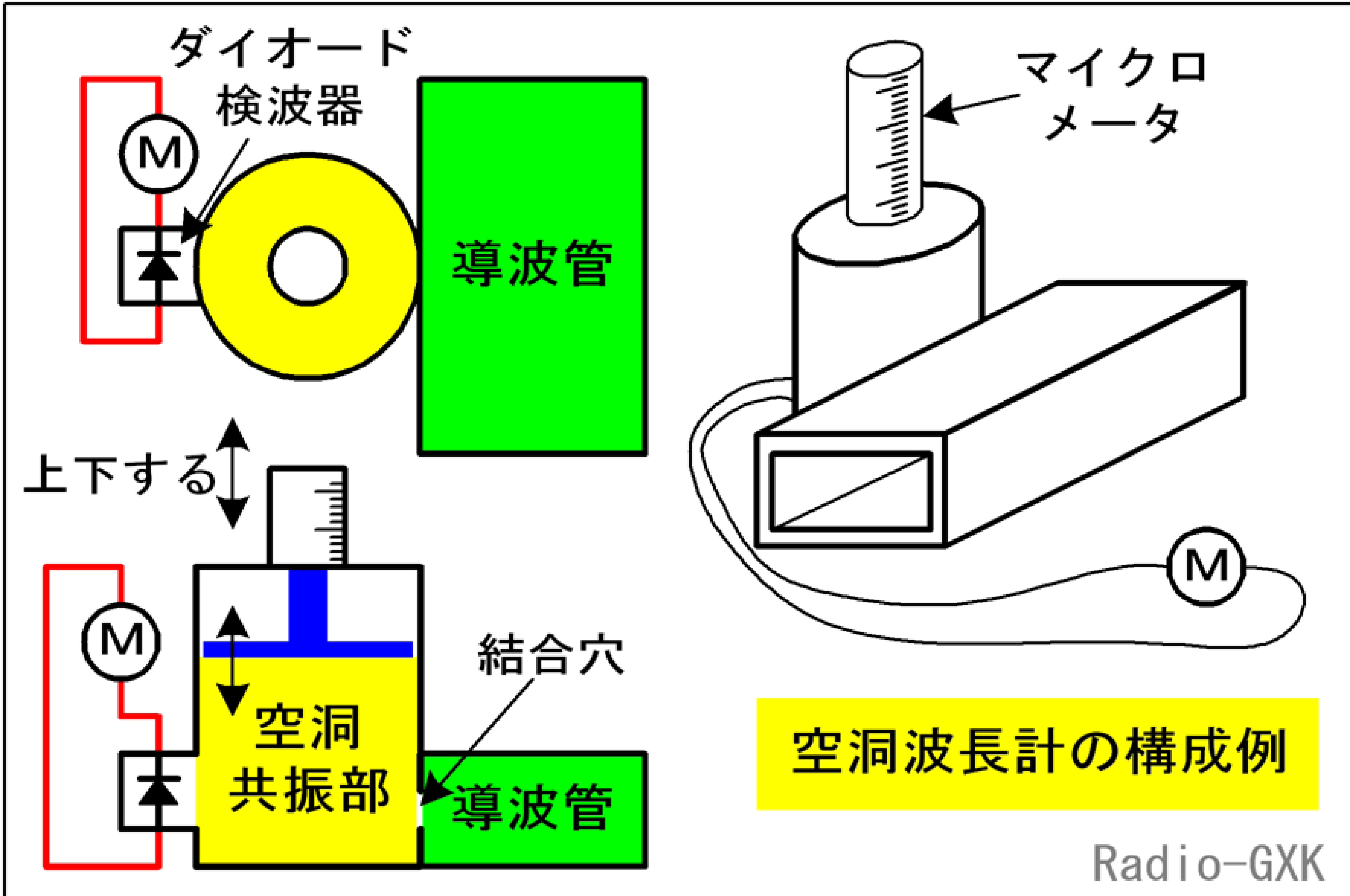
$g=5.6\text{x}10^{13}[s^{-1}]$, $\omega_{pe}=1.4\text{x}10^{16}\ [s^{-1}]$



*From Handbook of optical constants of solids edited by E.D.Palik

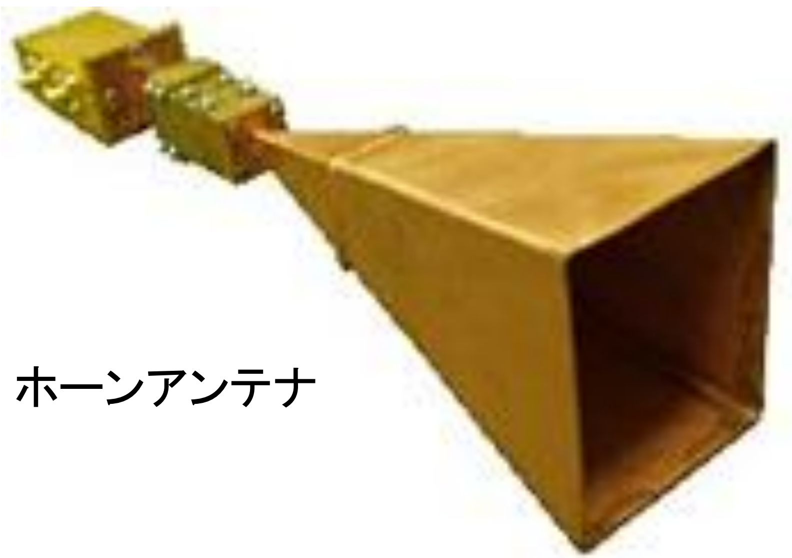
n=0.39, k=13.9 ⟺ n=2.4, k=3.59

応用例:空洞波長計

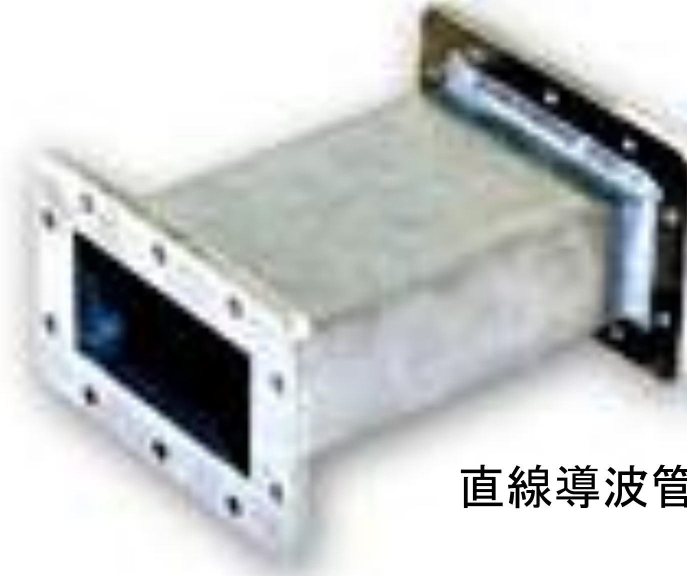


http://www.gxk.jp/elec/musen/1ama/H16/html/H1604A25_.html

マイクロ波導波管の実際



ホーンアンテナ



直線導波管

テーパー

高電力同軸導波管変換器

共振器の応用

- 共振によるフィルター
- 共振を利用した高電力化

光では？