電磁波工学X

米田仁紀

Dimension check

$$\langle P \rangle = \frac{1}{2} Re[E_z H_x] = -\frac{H_0 H*_0}{2\sigma\delta}$$

Power loss? => W??

$$I \cdot V = P$$ 電力＝電流×電圧

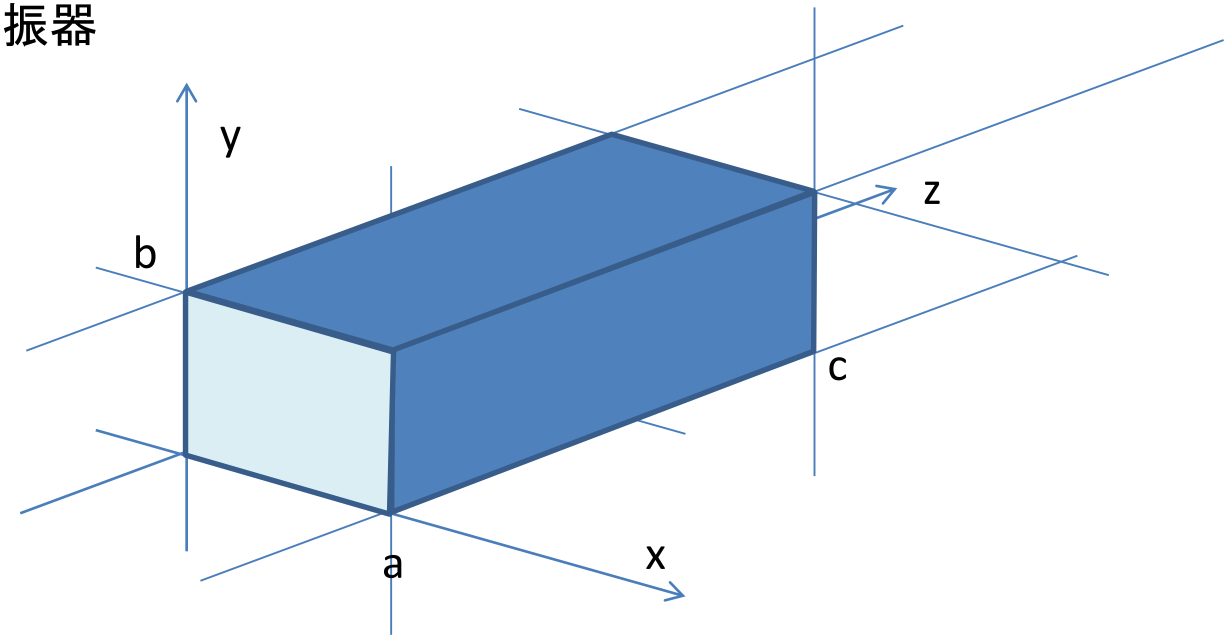
$$I = \frac{V}{Z}$$ オームの法則

$$\frac{E}{H} = Z$$ 電磁波のインピーダンス

$$H^2 = \frac{E^2}{Z^2} = \frac{V^2/m^2}{Z^2} = \frac{VI/m^2}{Z} = \frac{P}{Z}\frac{1}{m^2}$$

$$\frac{H^2}{\sigma\delta} = \frac{P}{Z}\frac{1}{m^2}\frac{1}{(\frac{1}{Zm}) \cdot m} = \frac{P}{m^2}$$

共振器



$$E_x = i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-ikz}$$

$$\Rightarrow i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{n\pi}{b}\right)H_0\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\left(Ae^{-ikz} + Be^{ikz}\right)$$

$$E_y = -i\frac{\omega\mu_0}{{k_0}^2 - k^2}\left(\frac{m\pi}{a}\right)H_0\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\left(Ae^{-ikz} + Be^{ikz}\right)$$

z=0, c で電場が0となる

$$A + B = 0$$
$$Ae^{-ikc} + Be^{ikc} = 0$$

$$e^{-ikc} - e^{ikc} = i2\sin kc = 0$$

$$k = l\frac{\pi}{c} \quad l = 1,2,3,\ldots$$

共振器

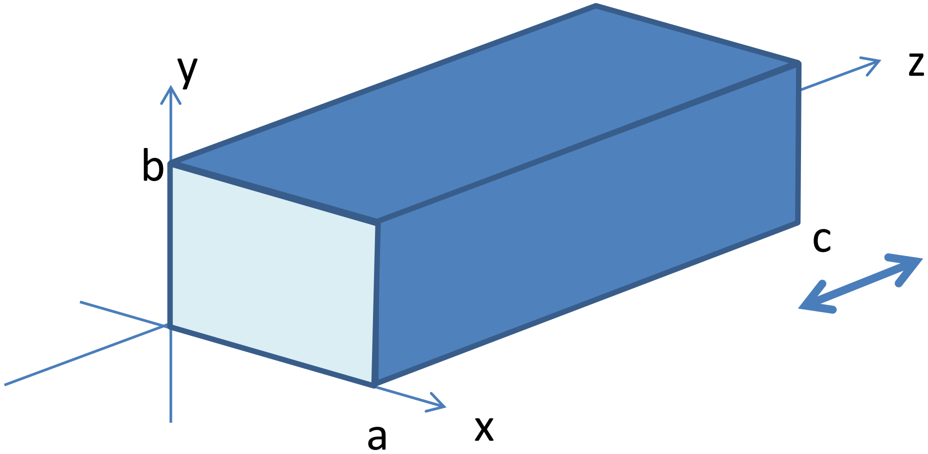
$$(\xi^2) + (\eta^2) = \left(k_0{}^2 - k^2\right)$$

$$(\xi^2) + (\eta^2) + k^2 = \left(k_0{}^2\right)$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2 = k_0{}^2$$

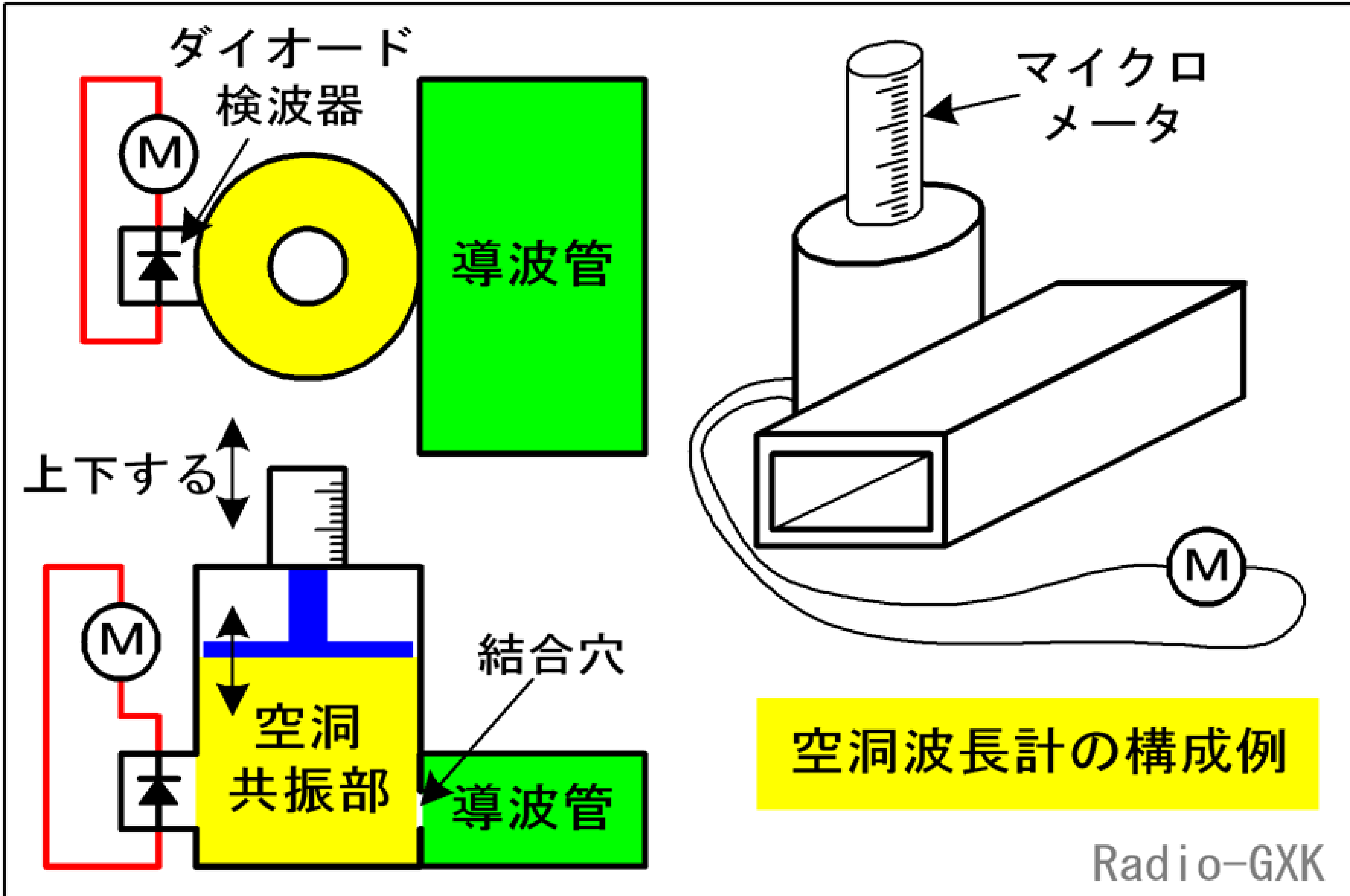
この時の周波数は

$$\omega_r = c\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{c}\right)^2}$$

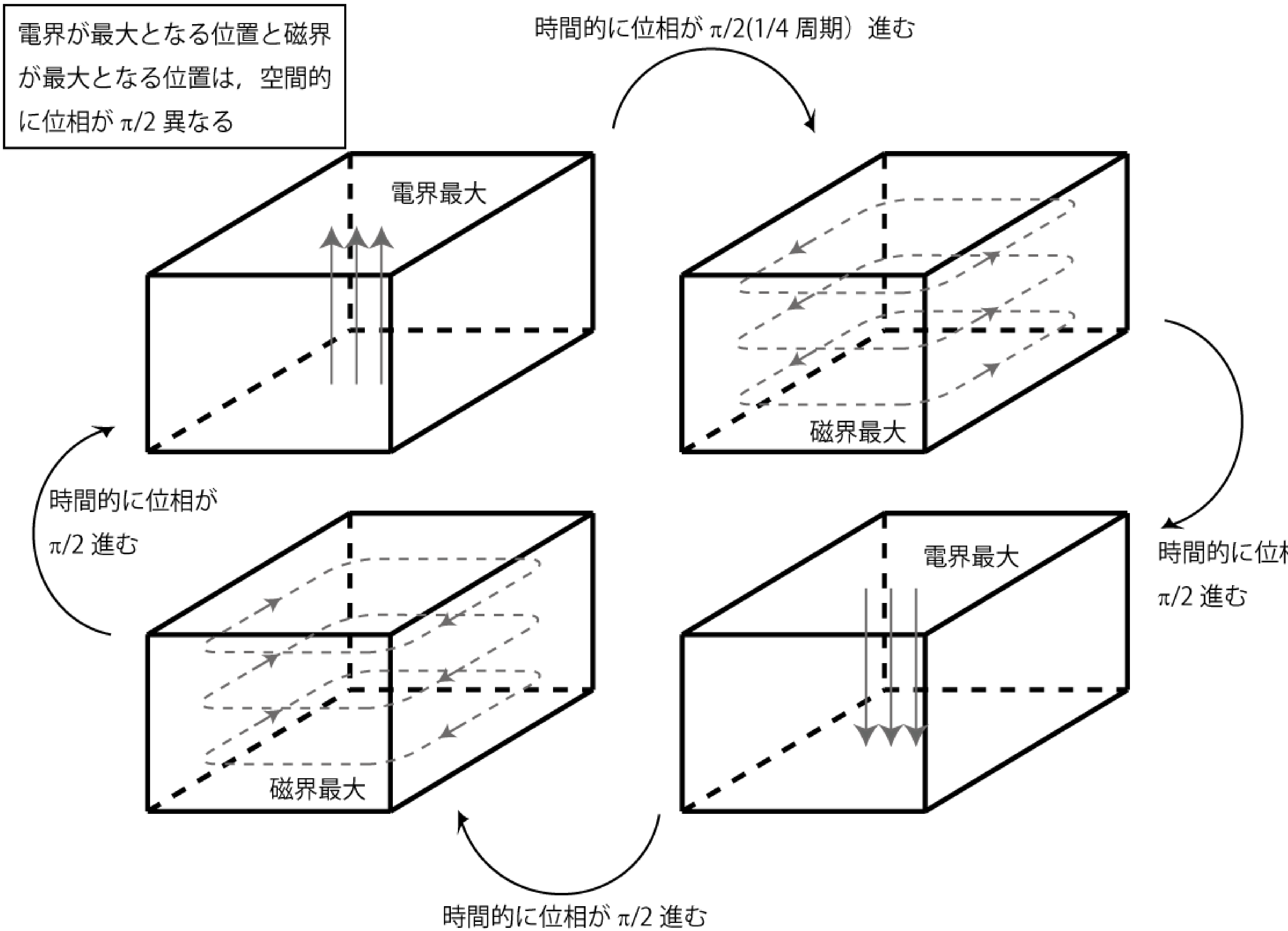


空洞波長計

前回

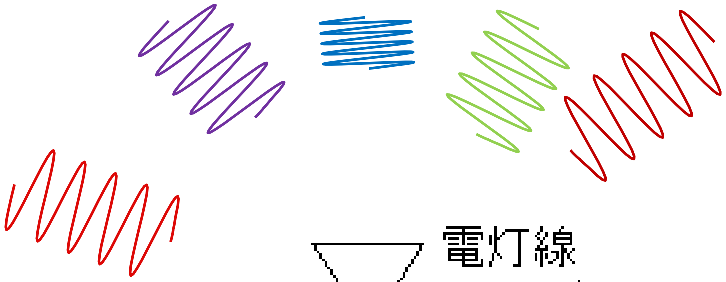


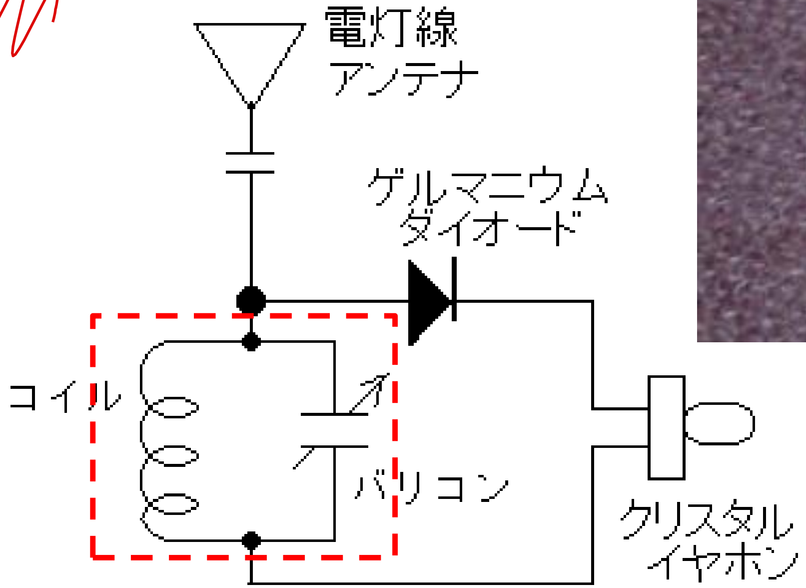
http://www.gxk.jp/elec/musen/1ama/H16/html/H1604A25_.html



電界が最大となる位置と磁界が最大となる位置は，空間的に位相が π/2 異なる
時間的に位相が π/2(1/4 周期）進む
電界最大
磁界最大
時間的に位相が π/2 進む
電界最大
磁界最大
時間的に位相が π/2 進む
時間的に位相が π/2 進む

そもそも共振とは



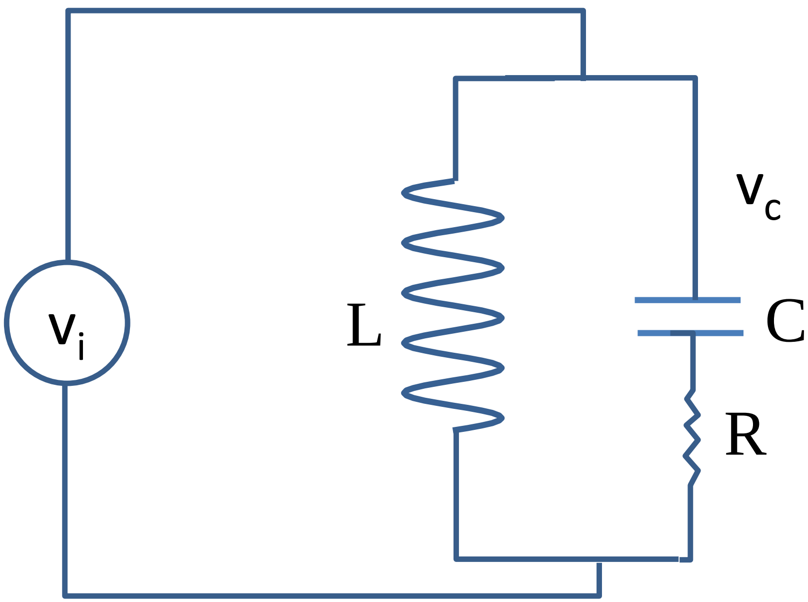


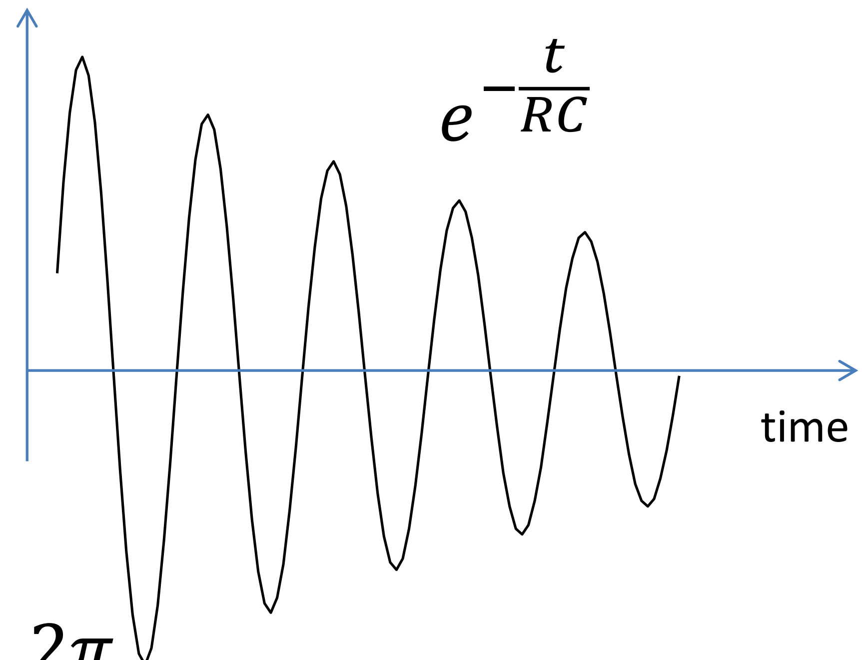
同調回路

共振を利用した周波数選択と増強

共振

系の持っている固有な振動数と外部の振動数が一致する。

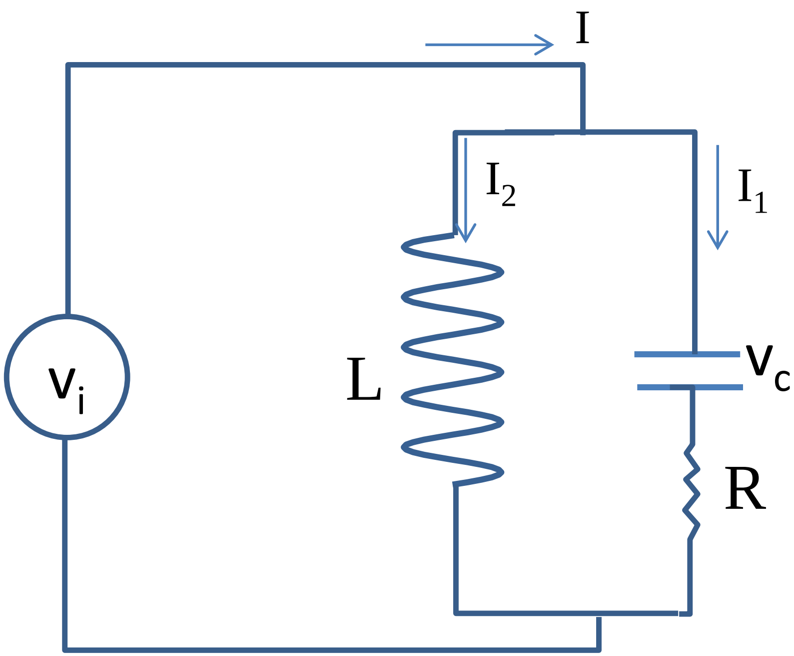




$$\omega_0 = \frac{2\pi}{\sqrt{LC}}$$

$$Q = 2\pi \frac{W_{em}}{W_{loss}}$$

<u>共振周波数とQ値が特性を決める</u>



$$\frac{1}{C}\int I_1 dt + RI_1 = L\frac{\partial I_2}{\partial t}$$

$$I = I_1 + I_2$$

$$I = I_0 e^{-i\omega t}$$

$$L\ddot{I_1} + R\dot{I_1} + \frac{1}{C}I_1 = -\omega^2 I_0 e^{-i\omega t}$$

$$L\ddot{I_1} + R\dot{I_1} + \frac{1}{C}I_1 = 0$$

$I_1 = Ae^{-i\omega_1 t}$として

$$-L\omega_1{}^2 - \omega_1 R + \frac{1}{C} = 0$$

$$\omega_1 = \frac{-i\frac{R}{L} \pm \sqrt{-\left(\frac{R}{L}\right)^2 + 4\frac{1}{LC}}}{2}$$

$$R^2 \ll \frac{4}{LC}$$

$$\omega_1 = \sqrt{\frac{1}{LC} - \left(\frac{R}{4L}\right)^2} - i\frac{R}{2L}$$

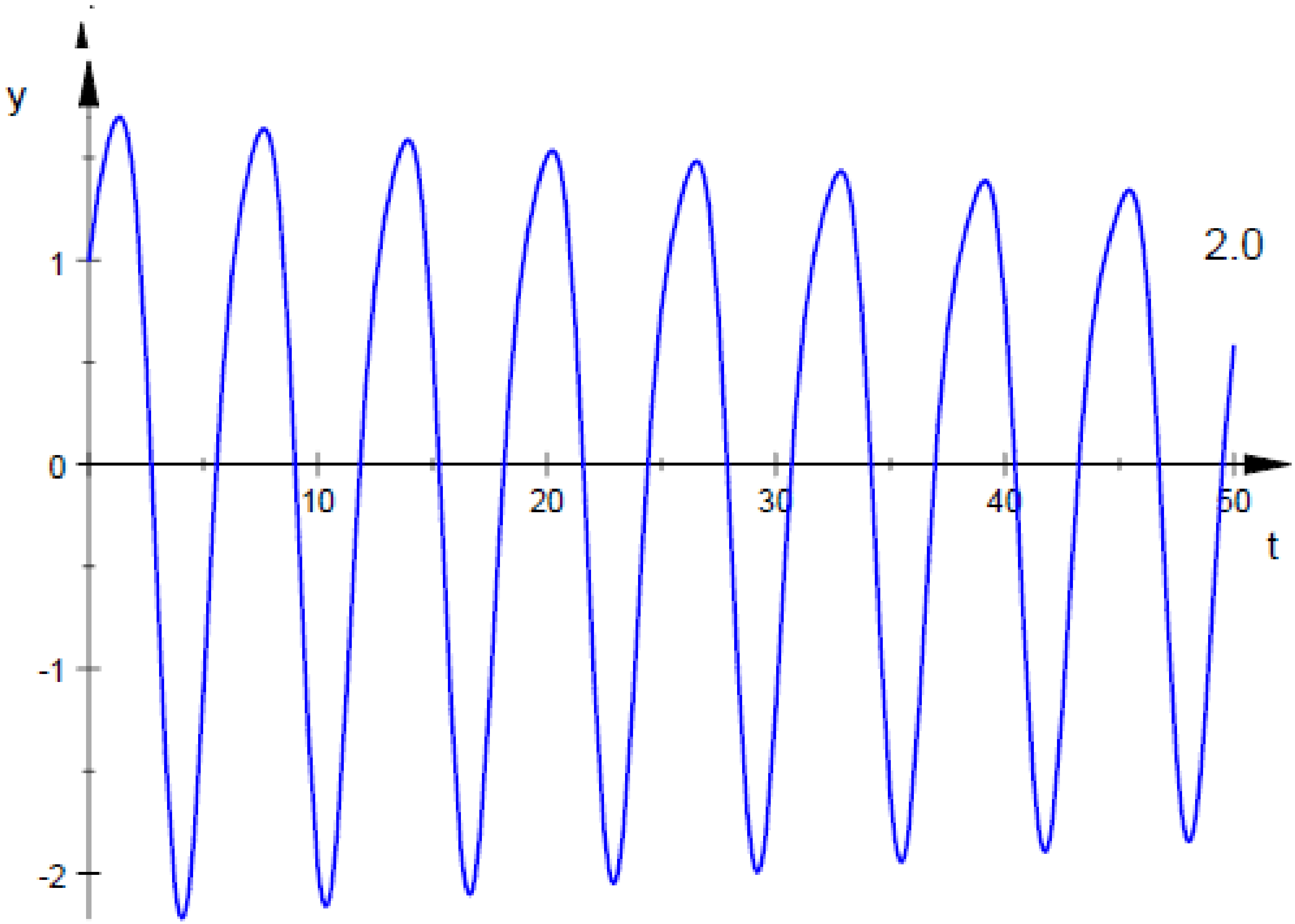
$$\omega_d - i\gamma$$

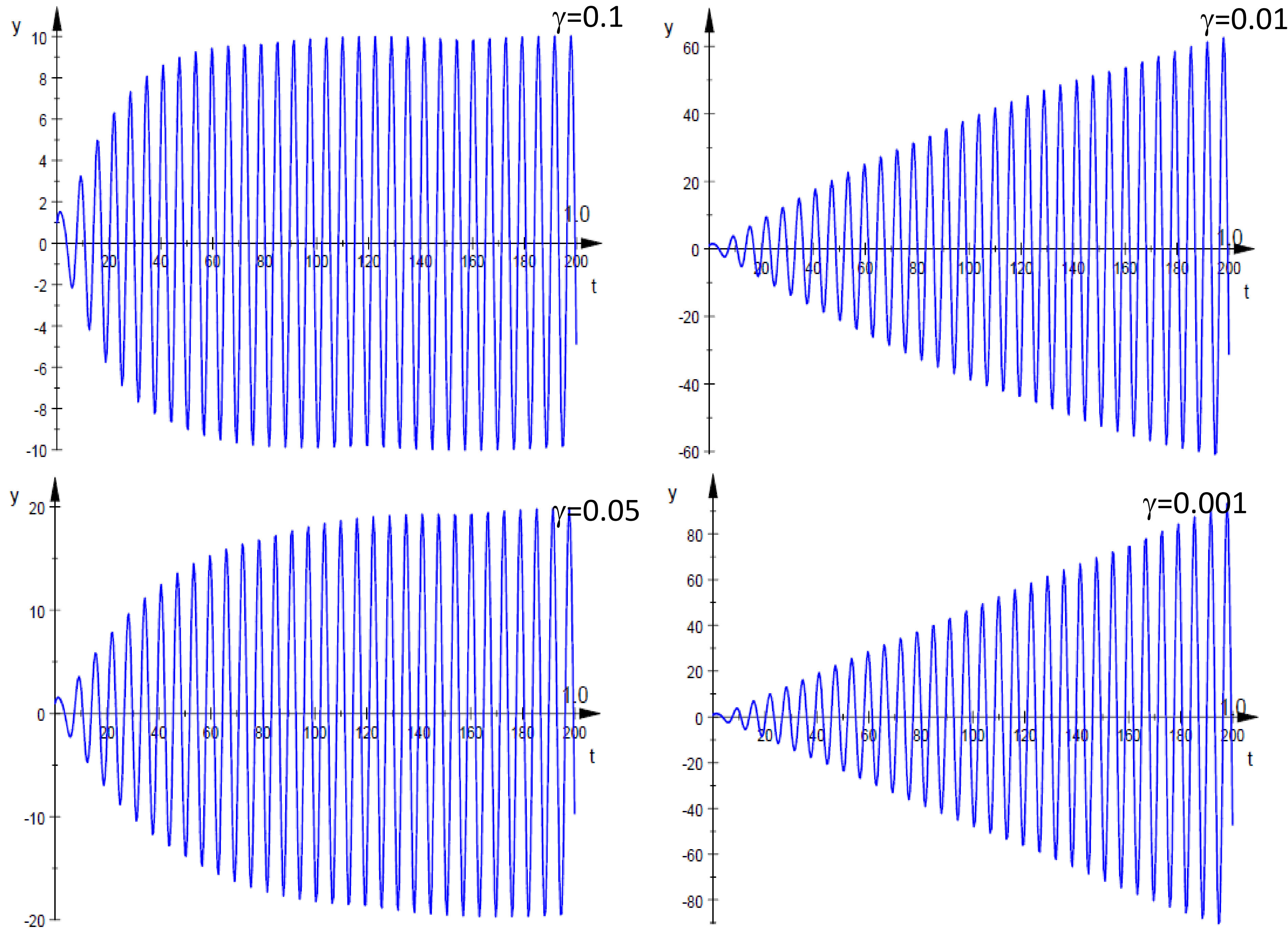
$$I_1 \approx \frac{1}{(\omega_1{}^2 - \omega^2)^2}$$

$$Q = \frac{\sqrt{1/LC}}{R/L} = \frac{\sqrt{\frac{L}{C}}}{R}$$

Derive with exp[-iωt]

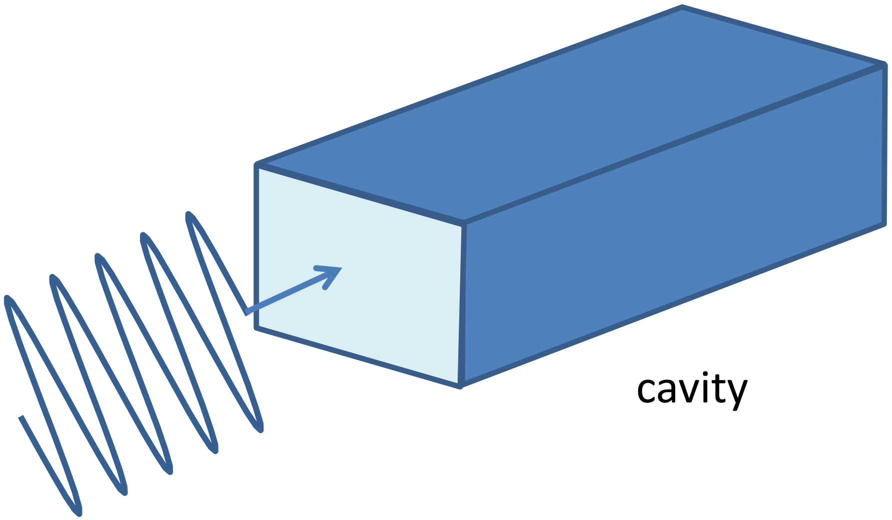
γ=0.01





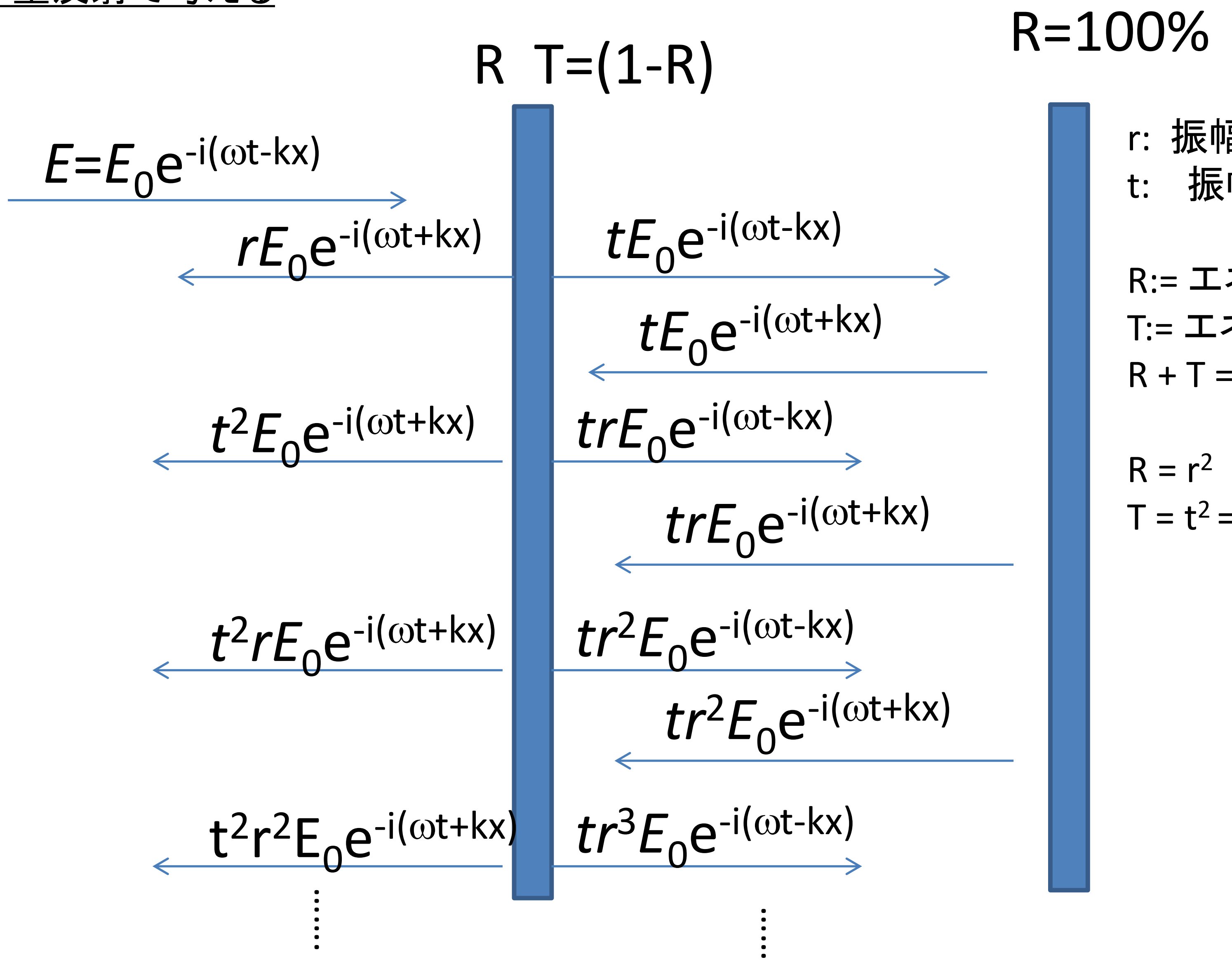
γ=0.1
γ=0.01
γ=0.05
γ=0.001
y
t
1.0

電磁波の場合も同じ



High Q cavity => Low loss => Perfect Wall(resistivity ~0)
=>high reflection=> difficult for input

多重反射で考える



r: 振幅反射率
t: 振幅透過率

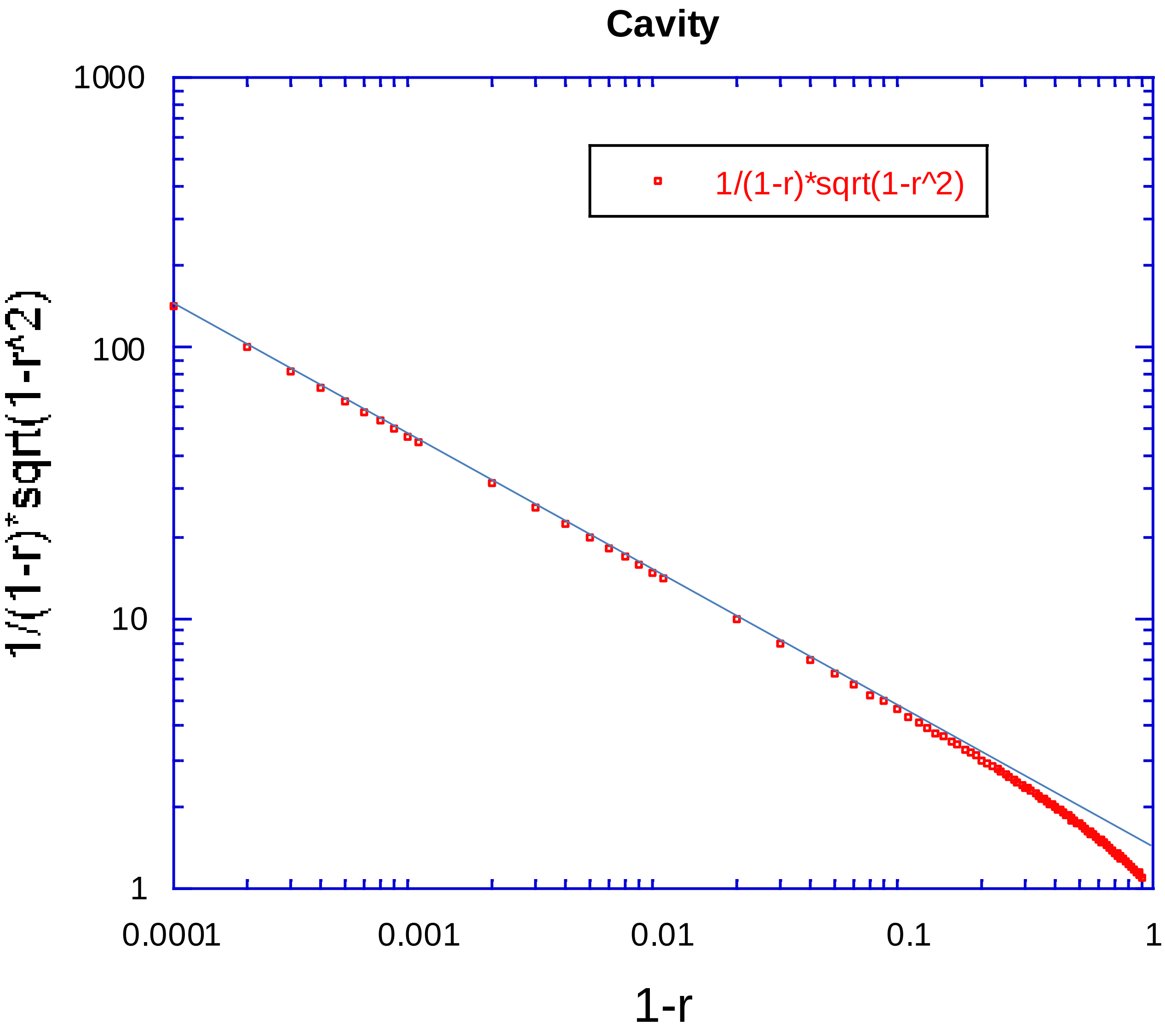
R:= エネルギー反射率
T:= エネルギー透過率
R + T = 1

$R = r^2$
$T = t^2 = 1-r^2$

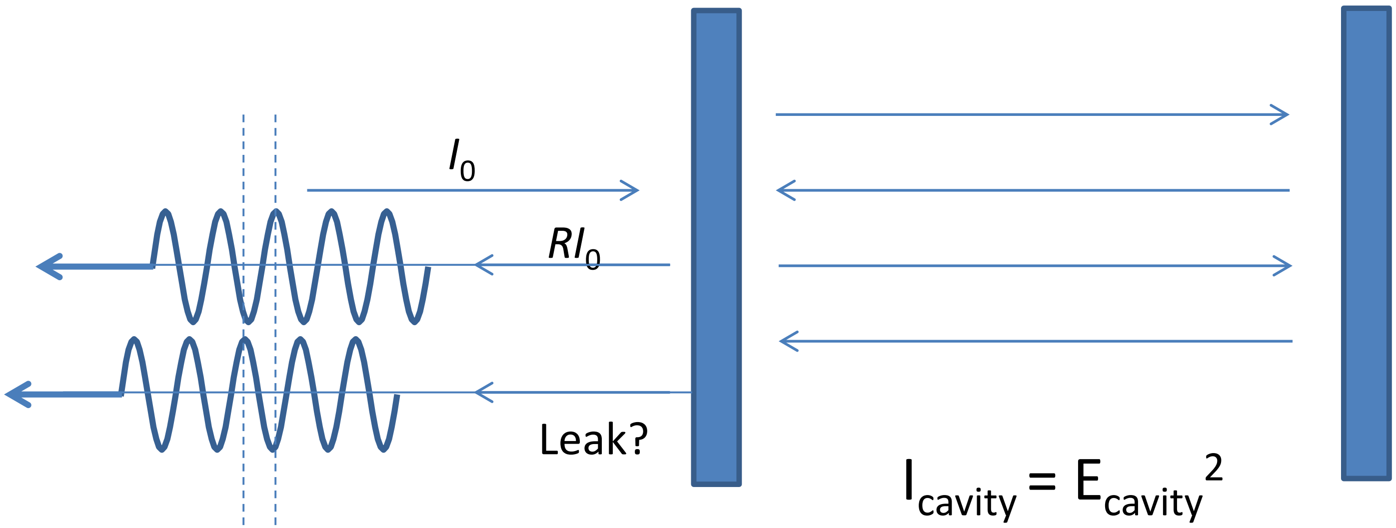
内部に入る総量は、全部がphaseがそろった場合

$$E_{cavity} = (1 + r + r^2 + r^3 \dots\dots)tE_0 e^{-i(\omega t - kx)} = \frac{1}{1-r} tE_0 e^{-i(\omega t - kx)}$$

$$\frac{\sqrt{1-r^2}}{1-r} = \frac{\sqrt{1+r}}{\sqrt{1-r}} \Rightarrow \frac{\sqrt{2}}{\sqrt{1-r}}$$



反射？

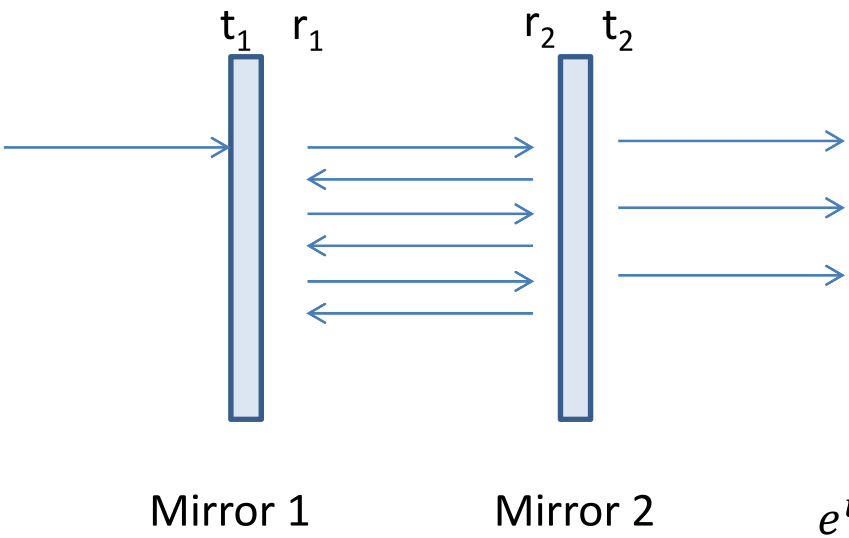


R~1 では、 $I_{cavity} = \frac{I_0}{1-r} = \frac{I_0}{1-\sqrt{R}}$

$$I_{leak} = (1 - R)I_{cavity} \Rightarrow RI_0$$

$$E_{cavity} = (1 + r + r^2 + r^3 \ldots \ldots)tE_0e^{-i(\omega t - kx)} = \frac{1}{1 - r}tE_0e^{-i(\omega t - kx)}$$

Fabry-Perot cavity



$$T(\nu) = \frac{I_t}{I_i} = \frac{T_{max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\frac{\pi\nu}{\nu_F})}$$

$$I_t = I_0(1 - r^2)^2 \left|\frac{1 - r^{2N} e^{i\delta N}}{1 - r^2 e^{i\delta}}\right|^2$$

$e^{i\delta}$は一周の位相

$$\delta = \frac{2\pi \cdot 2dn}{\lambda}$$

$N \Rightarrow \infty$では

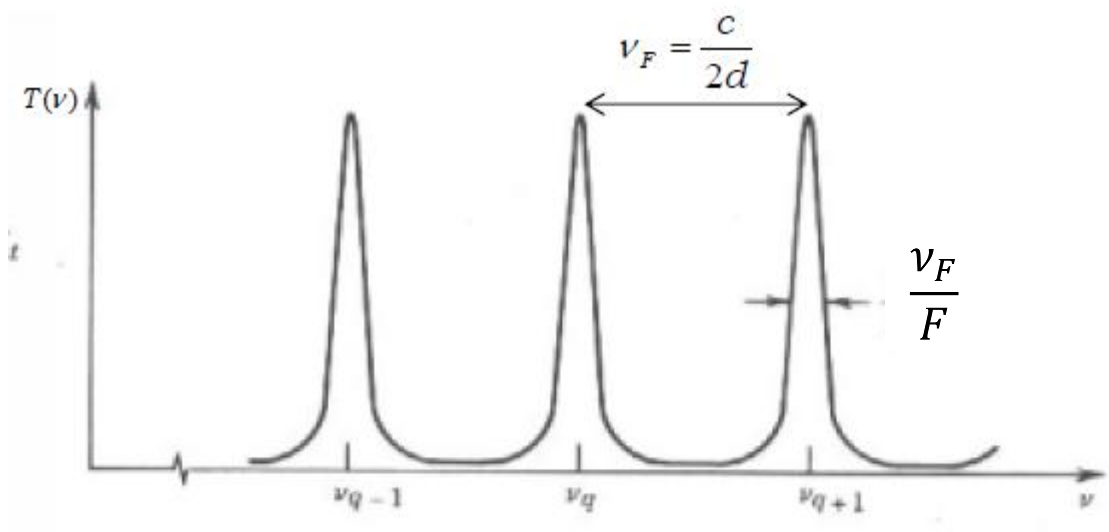
$$I_t = {I_0}^2 \left(\frac{(1-R)^2}{(1-R)^2 + 2R\left(2\sin^2\frac{\delta}{2}\right)}\right)$$

$$\frac{I_t}{I_0} = \left(\frac{(1-R)^2}{(1-R)^2 + 2R\left(2\sin^2\frac{\delta}{2}\right)}\right)$$

$$= \frac{1}{1 + \frac{4}{\pi^2}F^2\sin^2\frac{\delta}{2}}$$

$$Finesse; F = \frac{\pi\sqrt{R}}{1-R} = \frac{2\pi}{\Delta\delta_{1/2}}$$

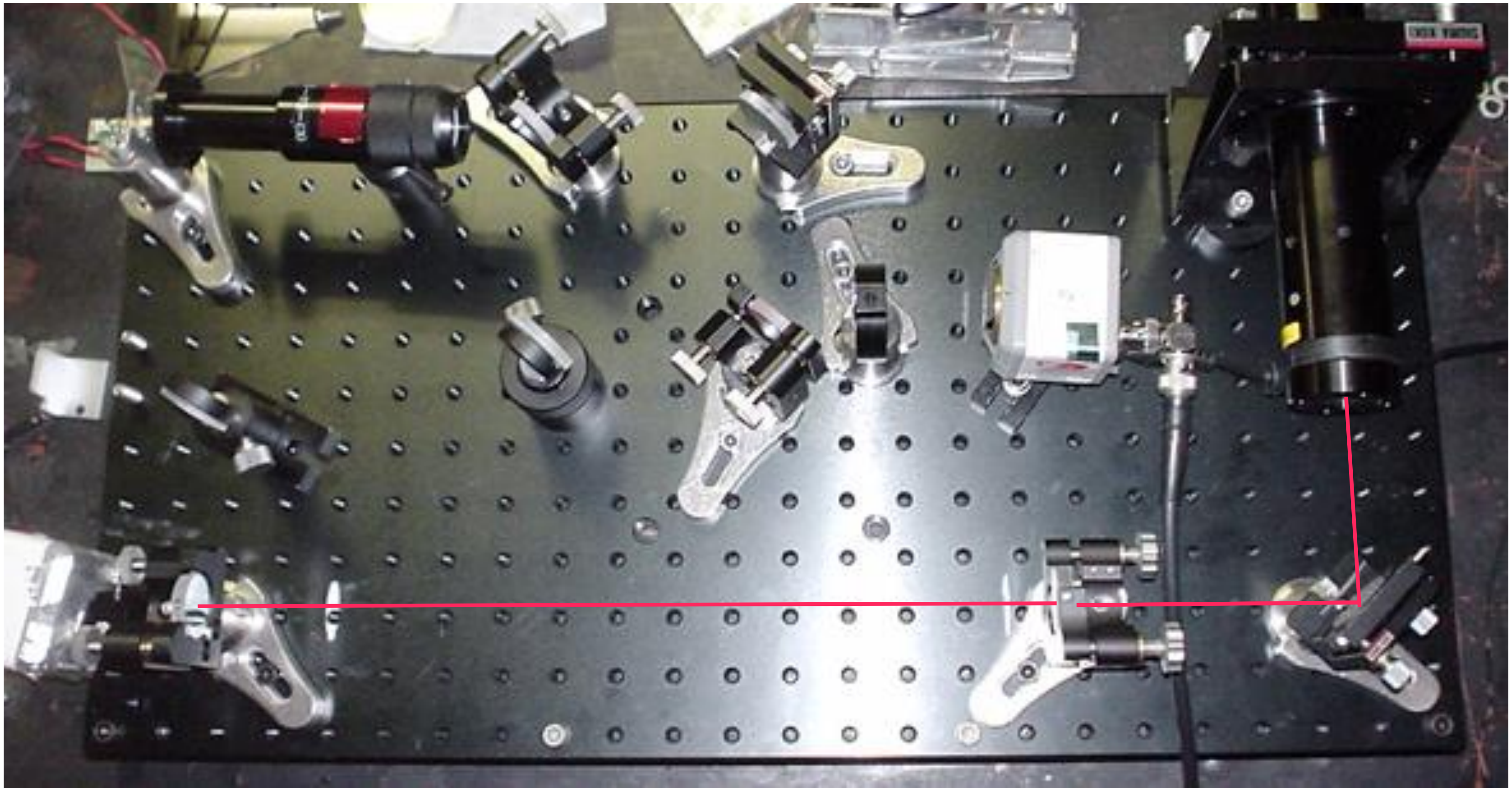
Free Spectral Range $\nu_F = \dfrac{c}{2d}$

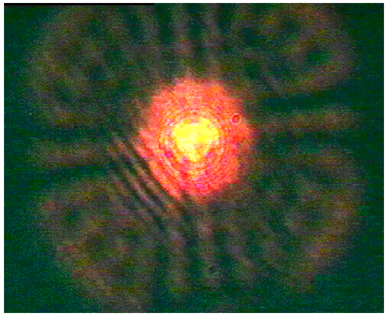
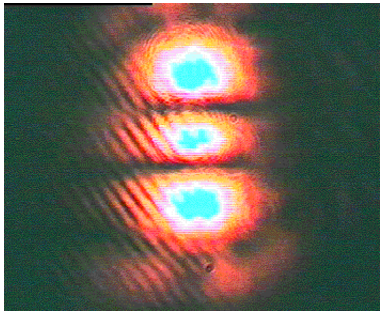
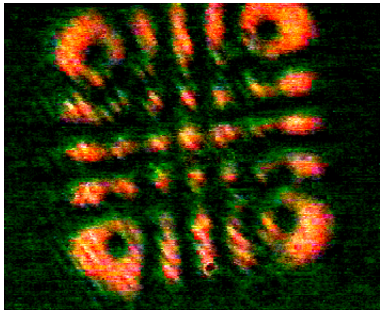
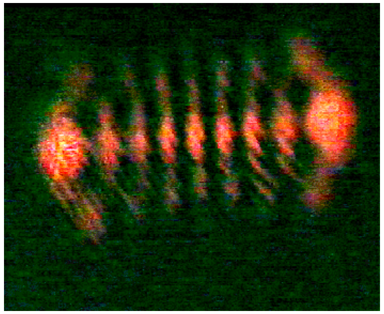


共振器のQ値は $Q = \dfrac{\nu_0}{\delta\nu} = \dfrac{\nu_0}{\nu_F}F$

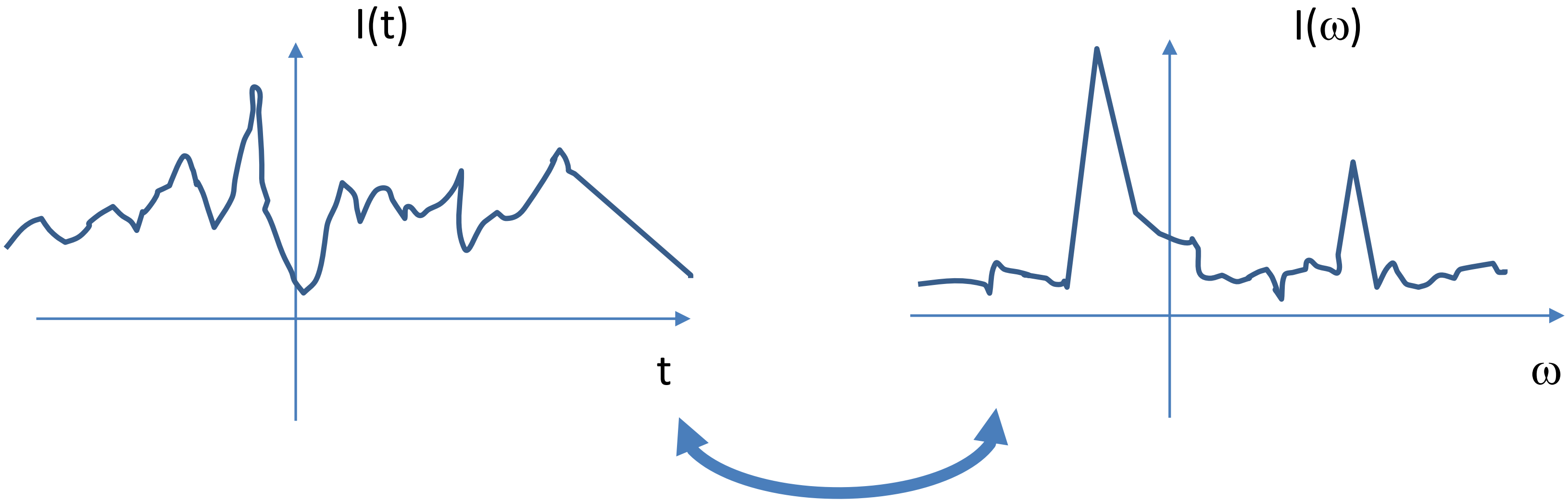
$$\text{Q値} = \frac{2\pi(\text{共振器に蓄えられたエネルギー})}{(\text{1周期で散逸するエネルギー})}$$

光を鏡と鏡の間に閉じ込めてみる



スペクトルの線幅（鋭さ）と寿命



フーリエ変換

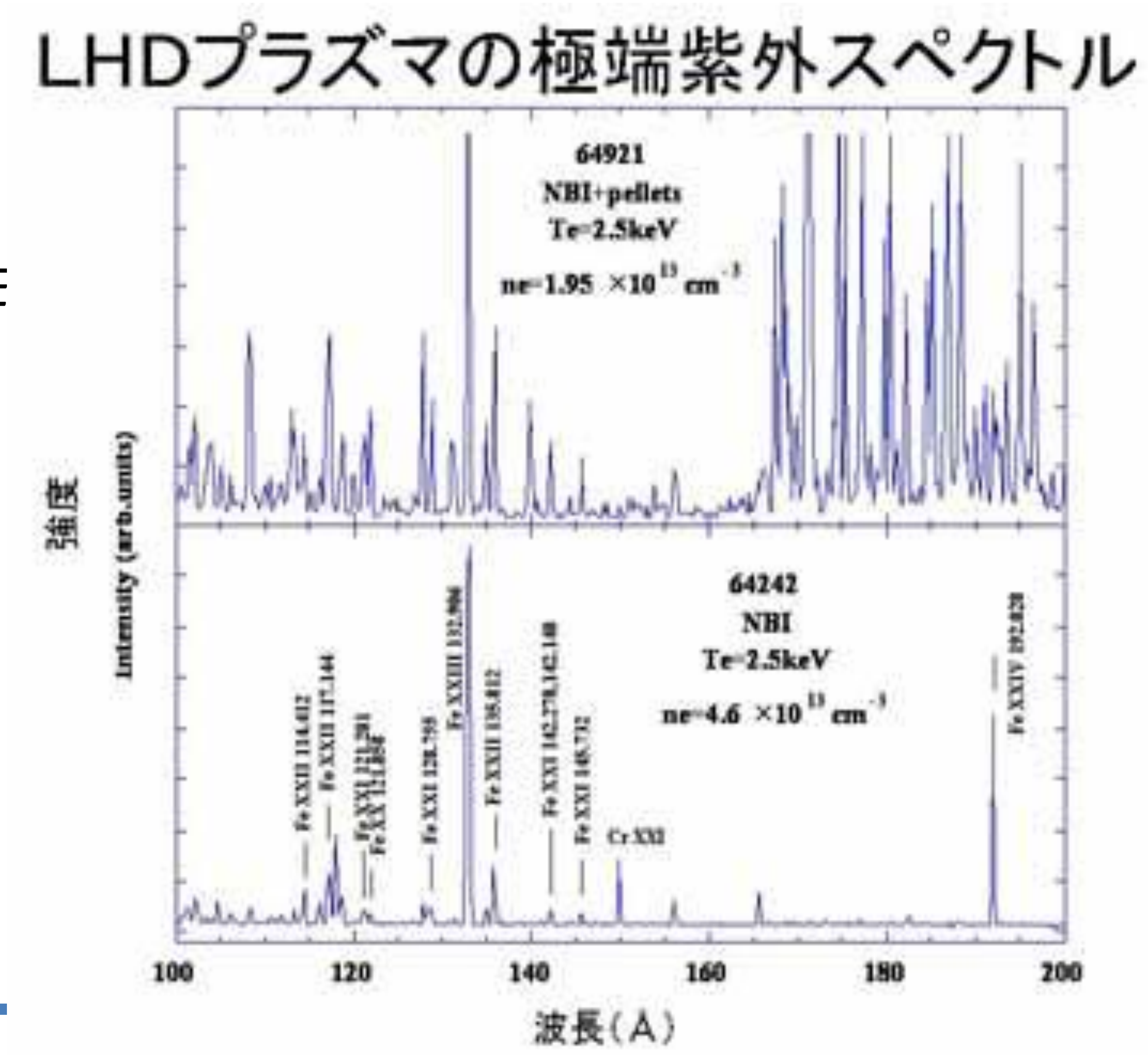
$$\int_{-\infty}^{\infty} E(t)e^{-i\omega t}dt = \tilde{E}(\omega) \qquad \int_{-\infty}^{\infty} \tilde{E}(\omega)e^{i\omega t}d\omega = E(t)$$

E(t) = const. ⇒ $\tilde{E}(\omega) = \delta(\omega)$

$E(t) = \delta(t)$ ⇐ $\tilde{E}(\omega) = const.$

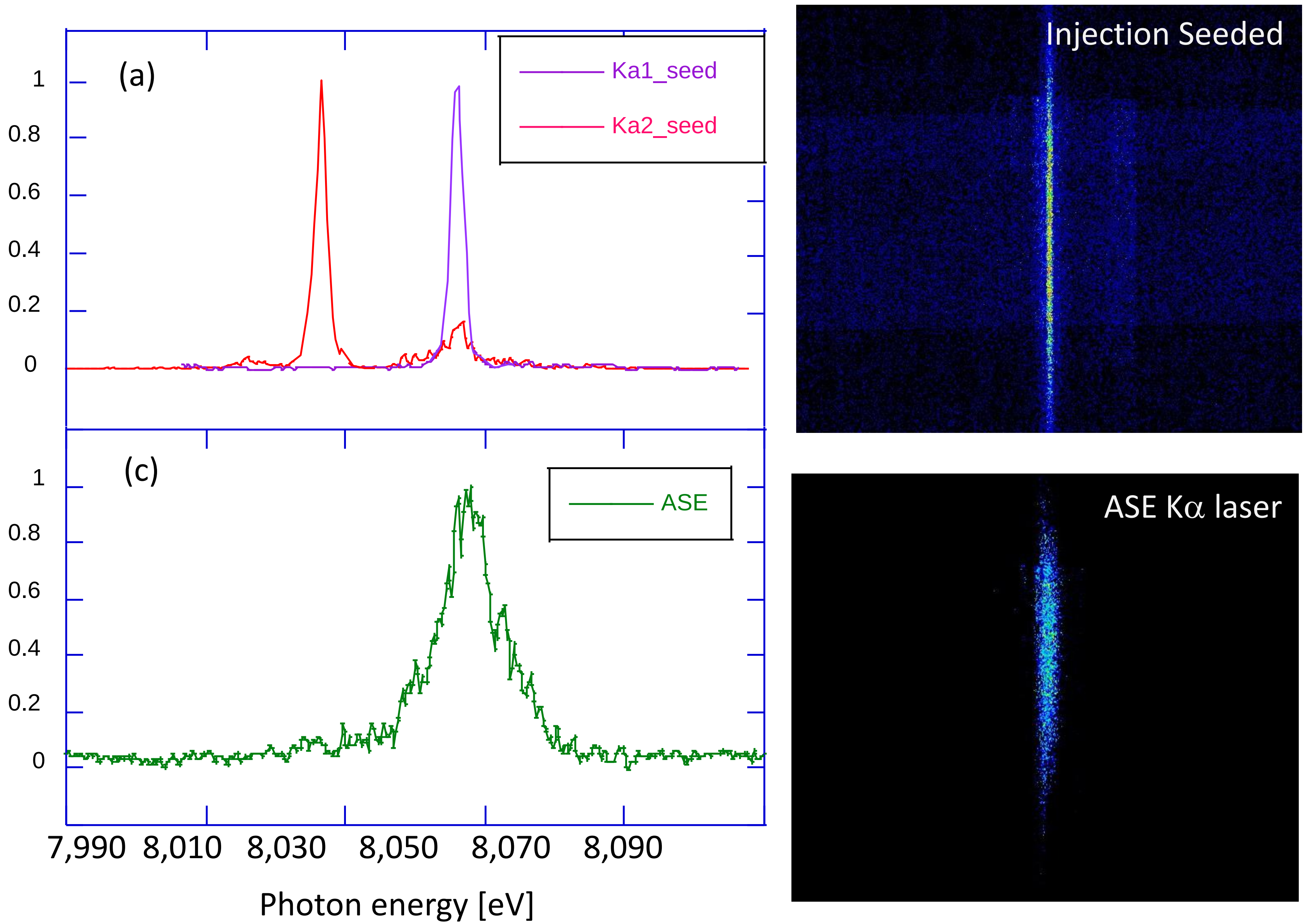
Energy

larger δE



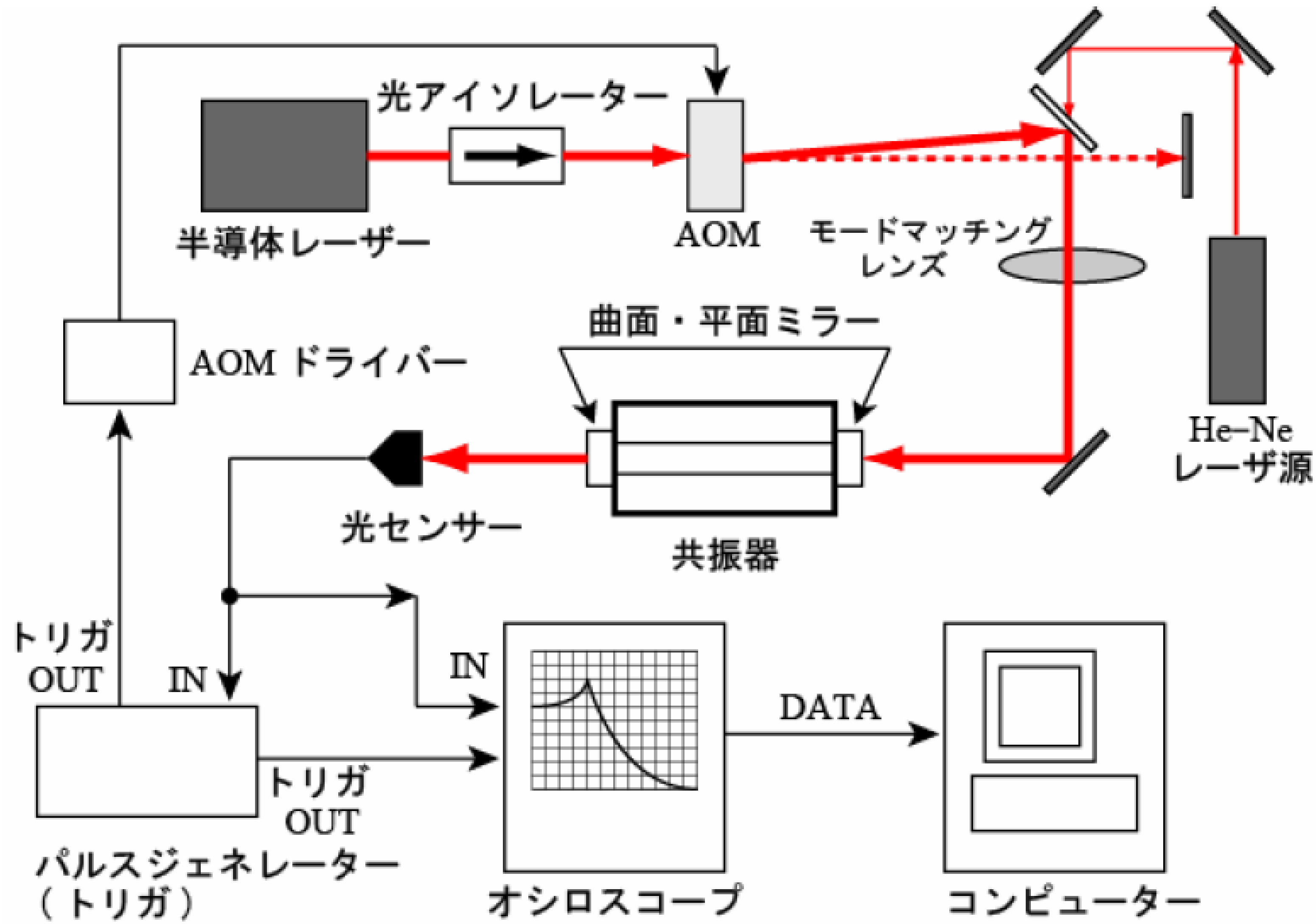
smaller δE

e

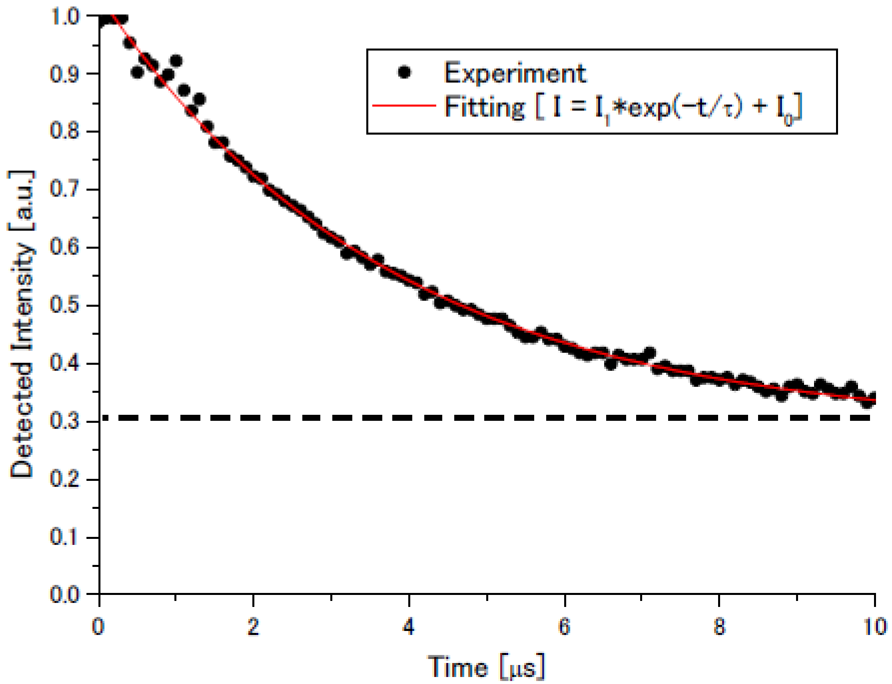


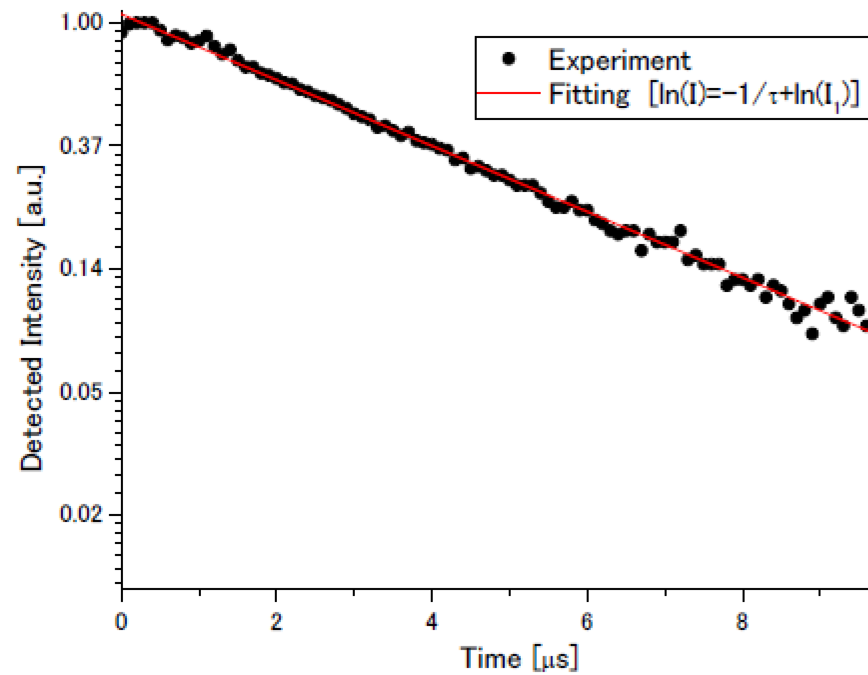
(a)
Ka1_seed
Ka2_seed
1
0.8
0.6
0.4
0.2
0
(c)
ASE
1
0.8
0.6
0.4
0.2
0
7,990
8,010
8,030
8,050
8,070
8,090
Photon energy [eV]
Injection Seeded
ASE Kα laser

Ring-down法による共振器の寿命測定



Ring-down法による測定結果





減衰時定数 $\tau = 3.6\ \mu s$

反射率 $R = 0.999982 \rightarrow F = 1.7 \times 10^5$, $Q = 4.4 \times 10^7$

透過率 14 ppm

損失 4 ppm （ミラー表面上における散乱、吸収)