

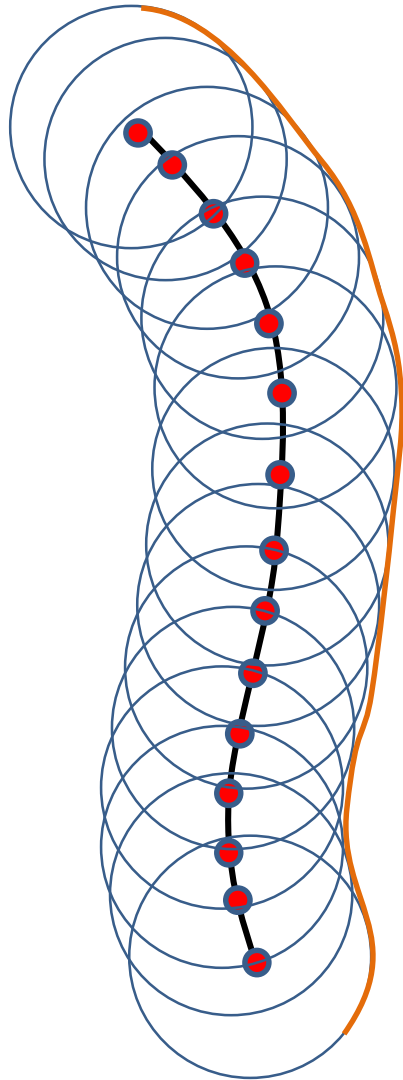
電磁波工学 XII

米田仁紀

電磁波の伝播

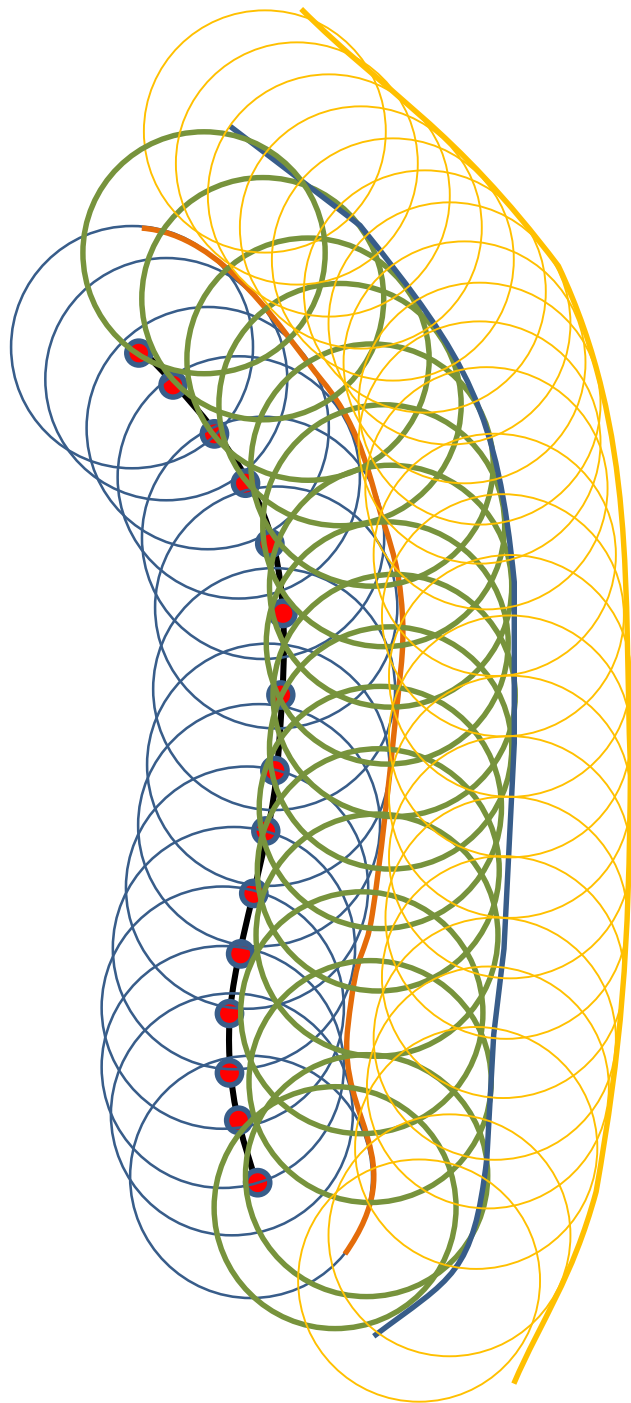
- ホイエンス・フレネルの原理
- 2つの回折(Fresnel Diffraction & Fraunhofer diffraction)
- フレネル積分
- 応用

ホイエンス・フレネルの原理



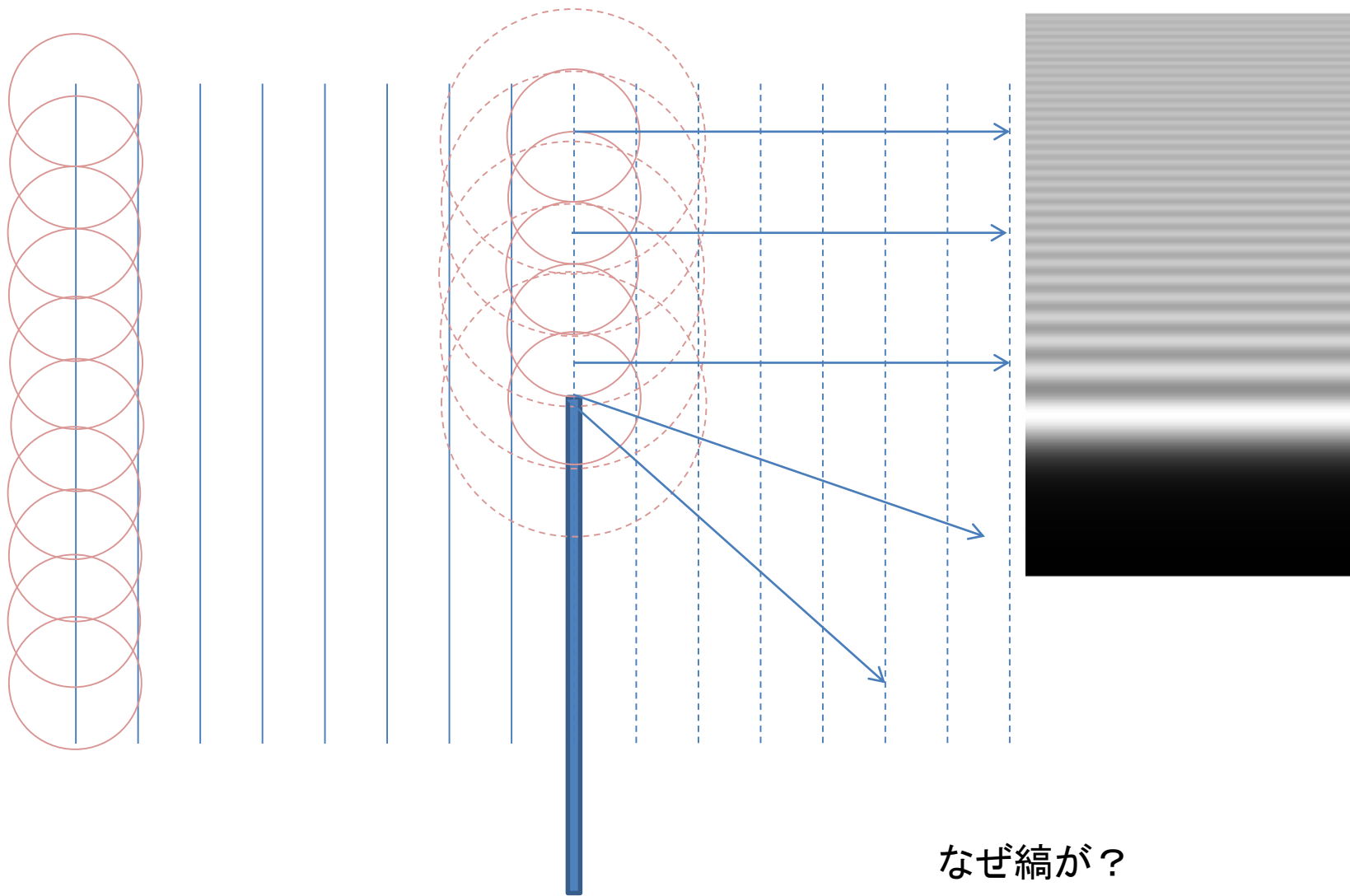
次の波面は、前の波面に点光源を置いた場合の重ね合わせとなる。

(決して後ろにはいかない。)



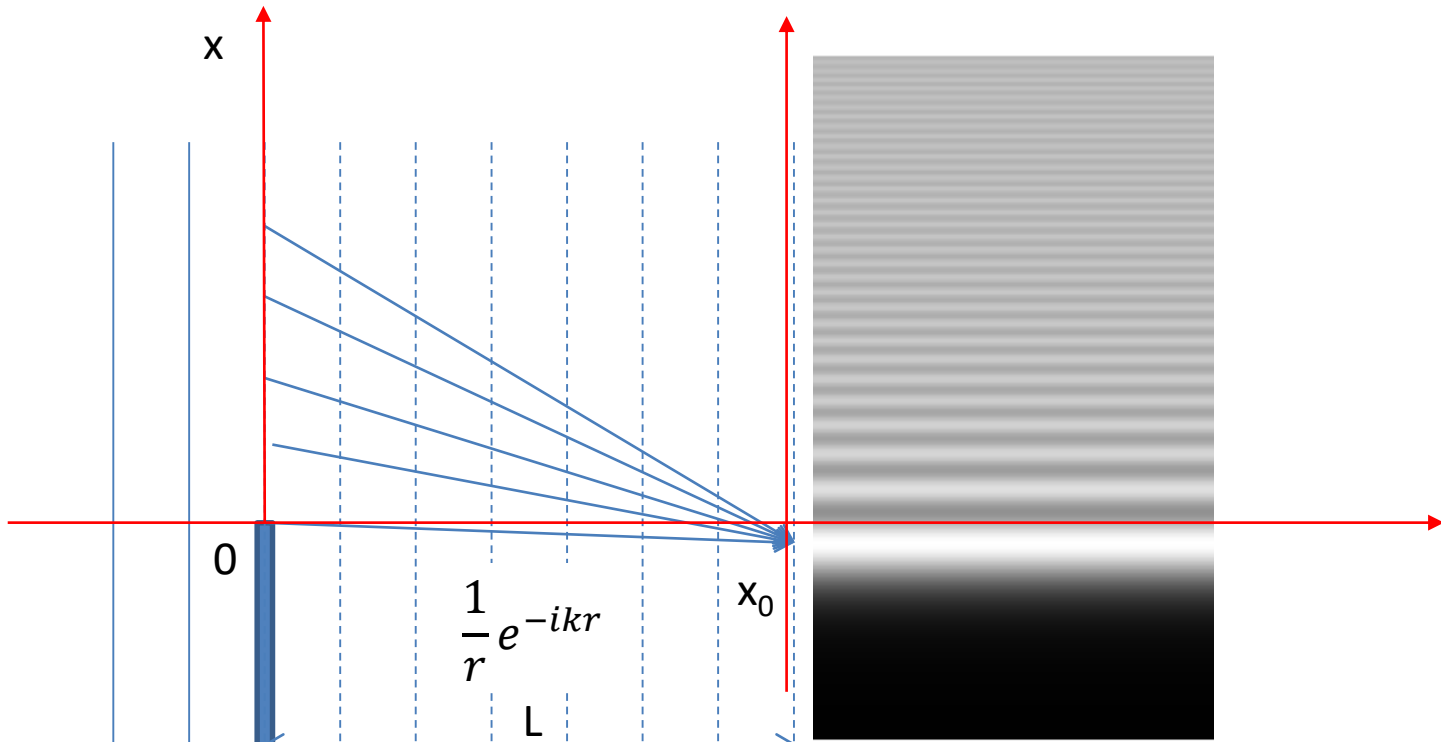
次々と点光源を仮定して進めばいい。

エッジの回折



なぜ縞が？

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$$E(x_0) = \int_0^\infty dx \frac{\exp[-ik\sqrt{L^2 + (x - x_0)^2}]}{\sqrt{L^2 + (x - x_0)^2}}$$

$$r = \sqrt{L^2 + (x - x_0)^2}$$

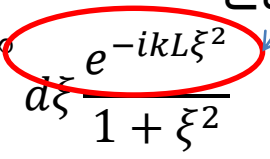
$$k = \frac{2\pi}{\lambda}$$

$$r = \sqrt{L^2 + (x - x_0)^2} = L \sqrt{1 + \frac{(x - x_0)^2}{L^2}} \approx L \left(1 + \left(\frac{x - x_0}{L} \right)^2 \right)$$

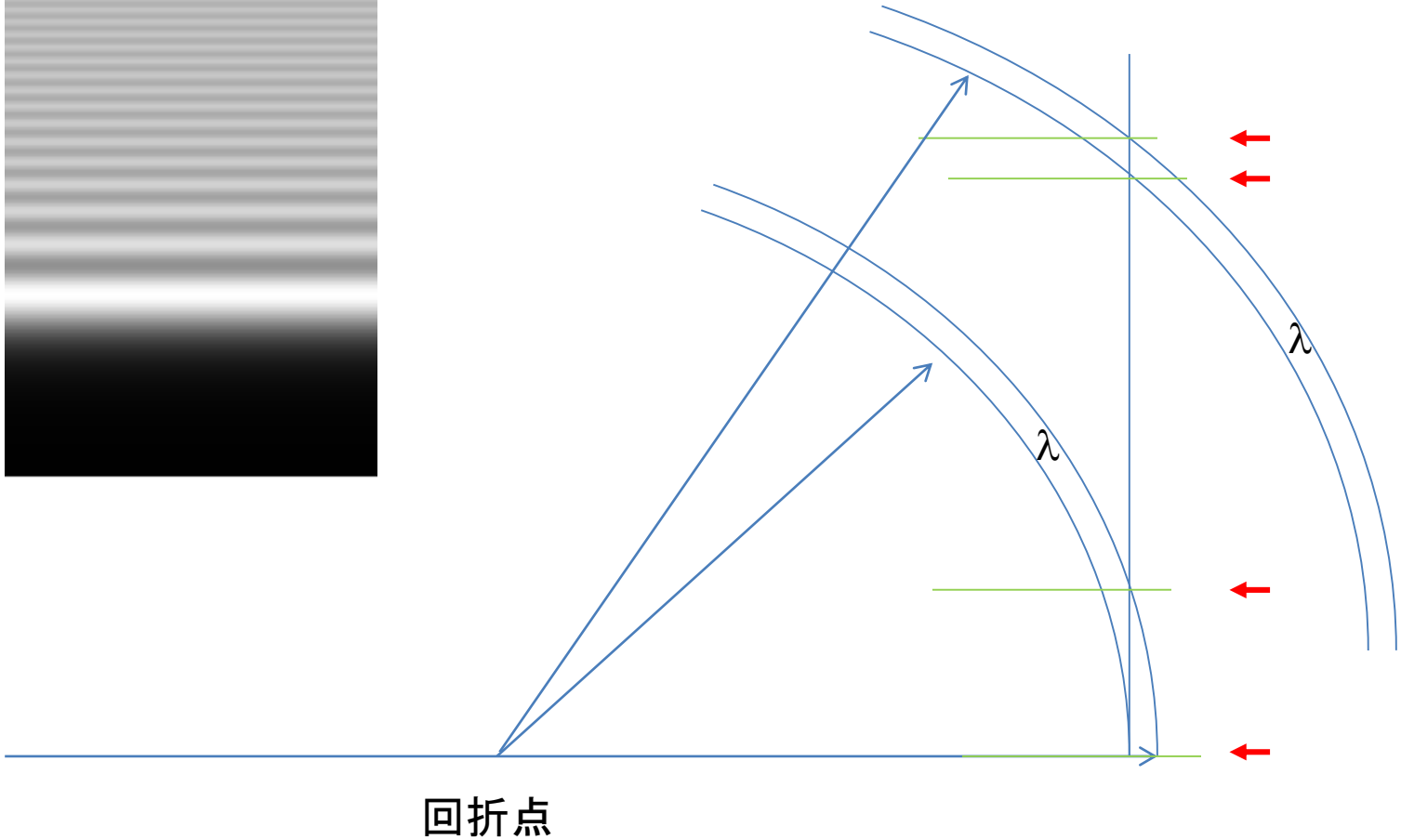
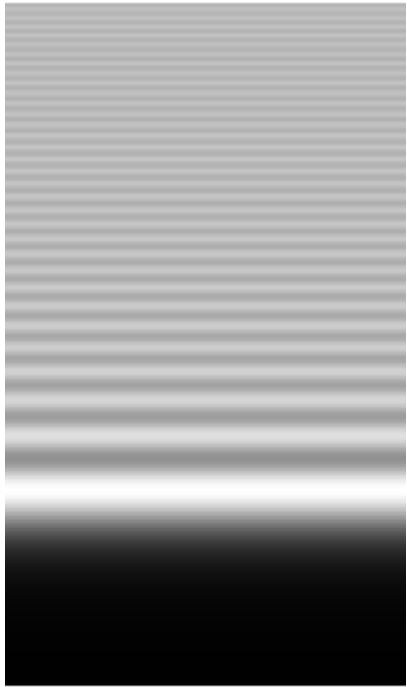
$$\xi = \frac{x - x_0}{L}$$

$$E(x_0) = \int_0^\infty dx \frac{\exp[-ik\sqrt{L^2 + (x - x_0)^2}]}{\sqrt{L^2 + (x - x_0)^2}} = e^{-ikL}/L \int_{\xi_0}^\infty d\xi \frac{e^{-ikL\xi^2}}{1 + \xi^2}$$

ここで振動



なぜ縞の間隔が狭まるか？



Fraunhofer diffraction

$$dE = \frac{E_A}{r} e^{i(\omega t - kr)} ds$$

$$r = \sqrt{X^2 + (Y - y)^2 + (Z - z)^2}$$

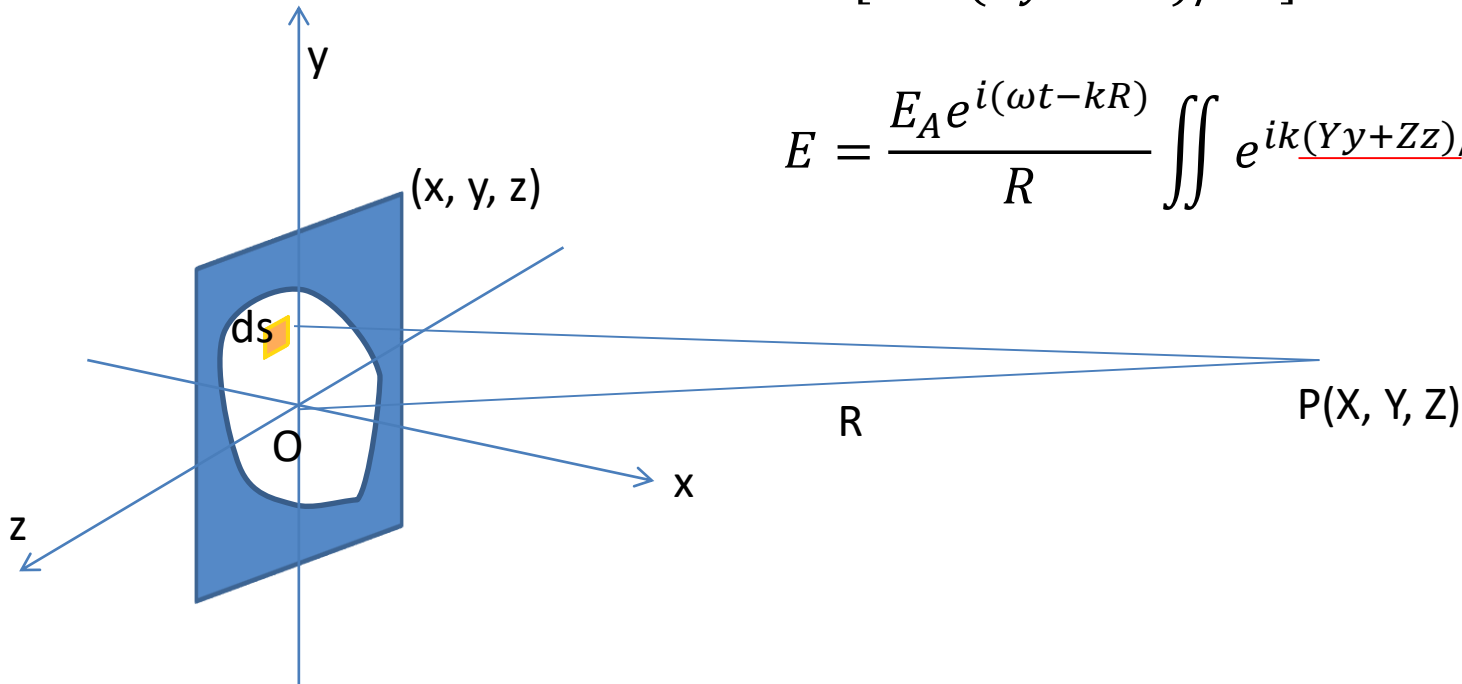
$$R = \sqrt{X^2 + Y^2 + Z^2}$$

$$y, z \ll R$$

$$r = R \sqrt{1 + (y^2 + z^2)/R^2 - 2(Yy + Zz)/R^2}$$

$$r \approx R [1 - (Yy + Zz)/R^2]$$

$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \iint e^{ik(Yy + Zz)/R} ds$$



Rectangular Aperture

$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \iint e^{ik(Yy + Zz)/R} ds$$



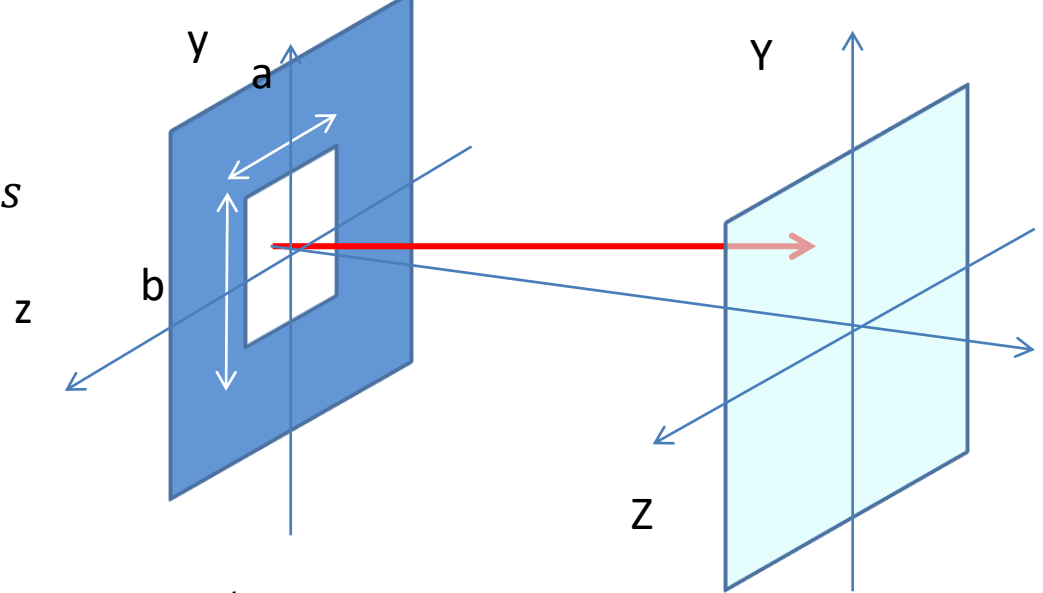
$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \int_{-b/2}^{b/2} e^{ikYy/R} dy \int_{-a/2}^{a/2} e^{ikZz/R} dz$$

ここで、 $\beta = \frac{kbY}{2R}$, $\alpha = \frac{kaZ}{2R}$ とすると

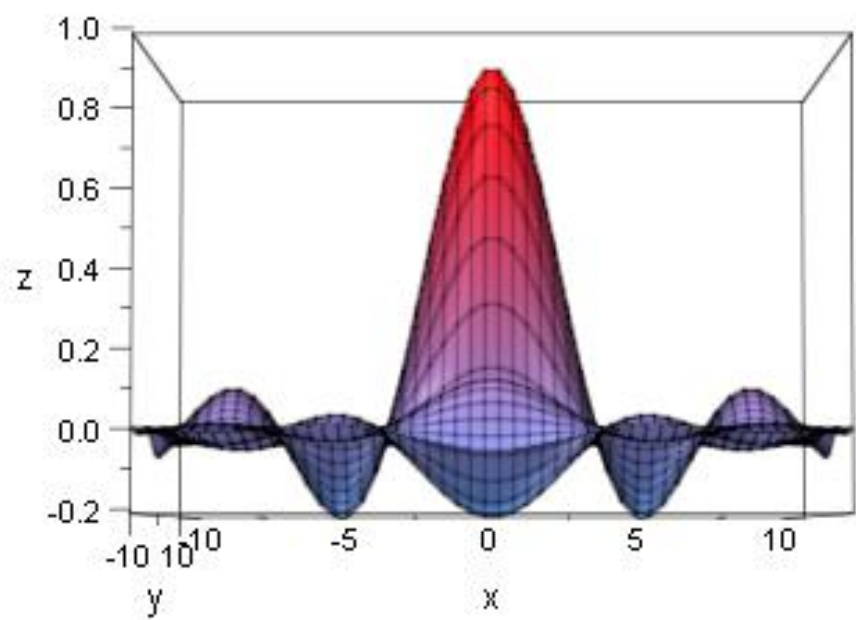
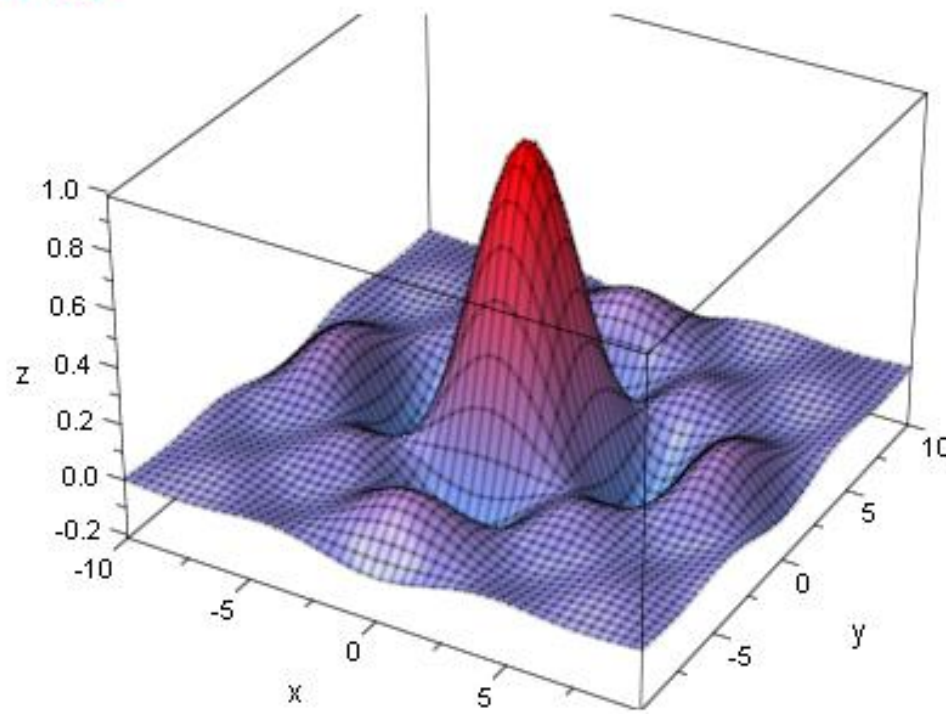
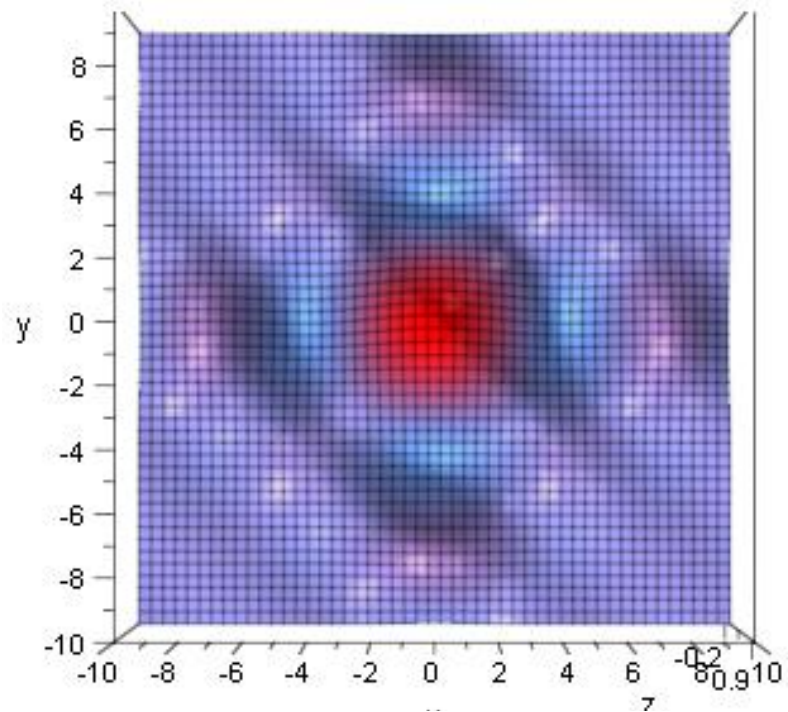
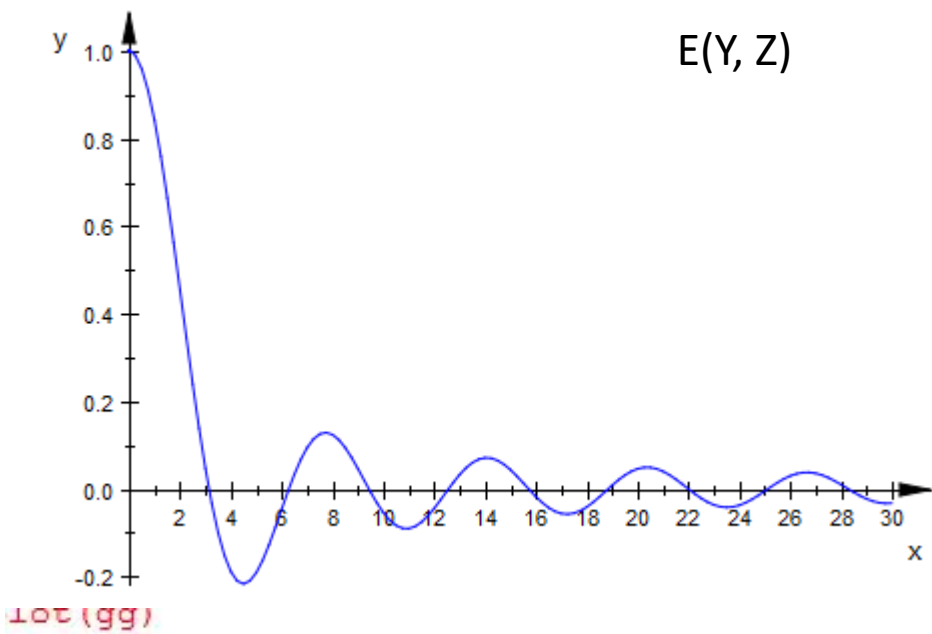
$$\int_{-b/2}^{b/2} e^{ikYy/R} dy = b \left(\frac{e^{i\beta} - e^{-i\beta}}{2i\beta} \right) = b \left(\frac{\sin\beta}{\beta} \right)$$

$$E = \frac{AE_A e^{i(\omega t - kR)}}{R} \left(\frac{\sin\alpha}{\alpha} \right) \left(\frac{\sin\beta}{\beta} \right)$$

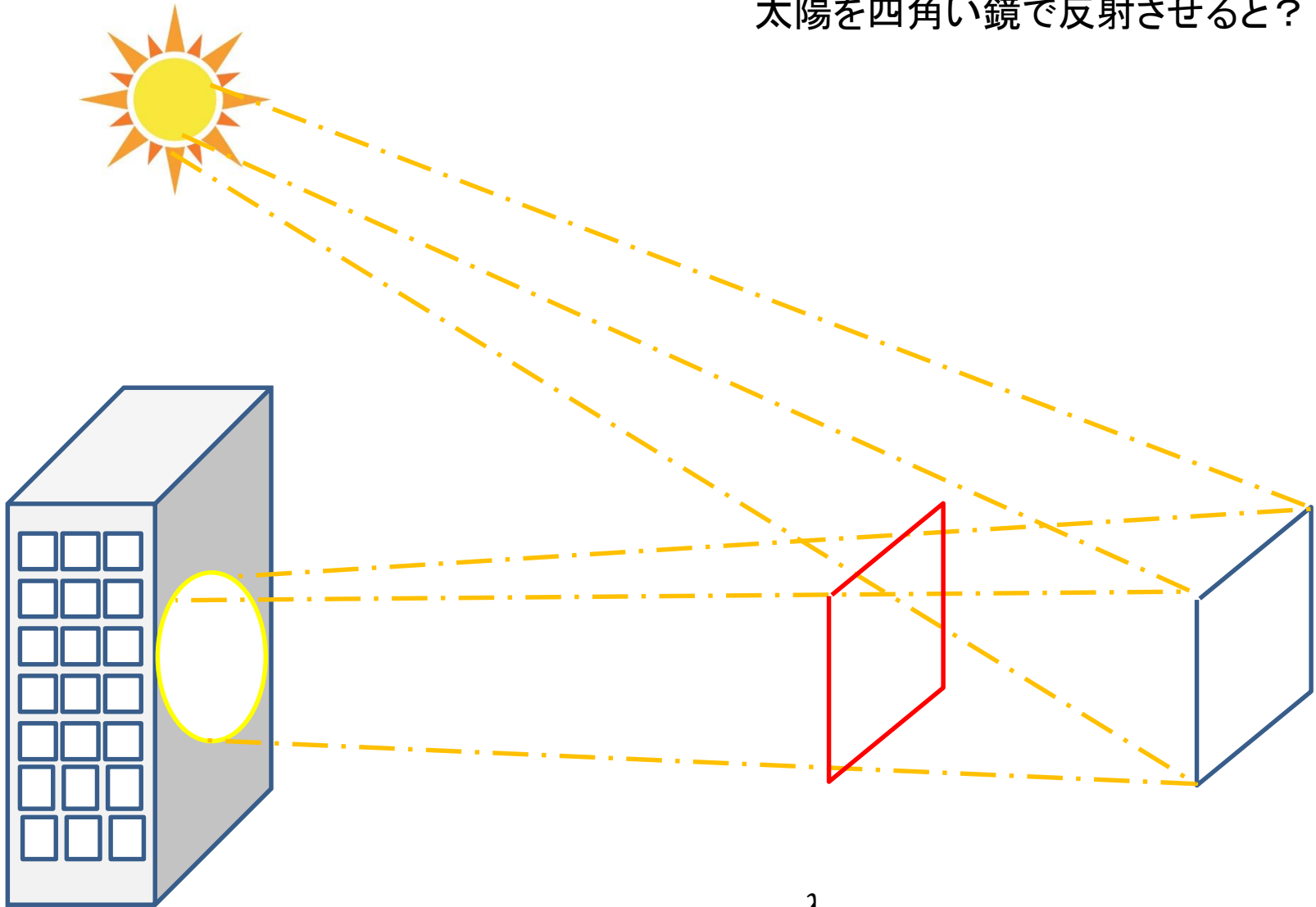
$$I(Y, Z) = I(0) \left(\frac{\sin\alpha}{\alpha} \right)^2 \left(\frac{\sin\beta}{\beta} \right)^2$$



$E(Y, Z)$



太陽を四角い鏡で反射させると？



$$D = L\theta = L \frac{\lambda}{d}$$

$L = 20\text{m}, d = 0.05\text{m}, \lambda = 0.5\mu\text{m}$
 $D = 2\text{mm}$

Circular Aperture

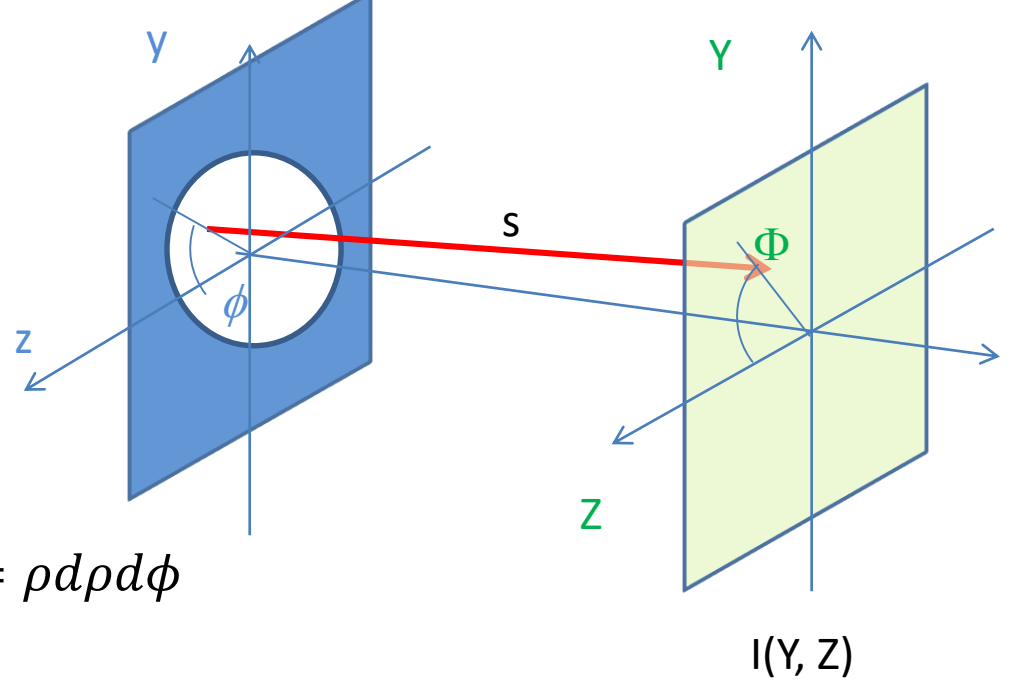
$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \iint e^{ik(Yy + Zz)/R} ds$$



$$z = \rho \cos \phi, y = \rho \sin \phi$$

$$Z = q \cos \Phi, Y = q \sin \Phi$$

$$ds = \rho d\rho d\phi$$



$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \int_{\rho=0}^a \int_{\phi=0}^{2\pi} e^{i\left(\frac{k\rho q}{R}\right) \cos(\phi - \Phi)} \rho d\rho d\phi$$

$\Phi = 0$ として計算

$$J_0(u) = \frac{1}{2\pi} \int_0^{2\pi} e^{iu \cos u} du \quad u = \frac{k\rho q}{R} \quad \text{Bessel function}$$

$$E = \frac{E_A e^{i(\omega t - kR)}}{R} 2\pi \int_0^a J_0\left(\frac{k\rho q}{R}\right) \rho d\rho$$

Bessel Function

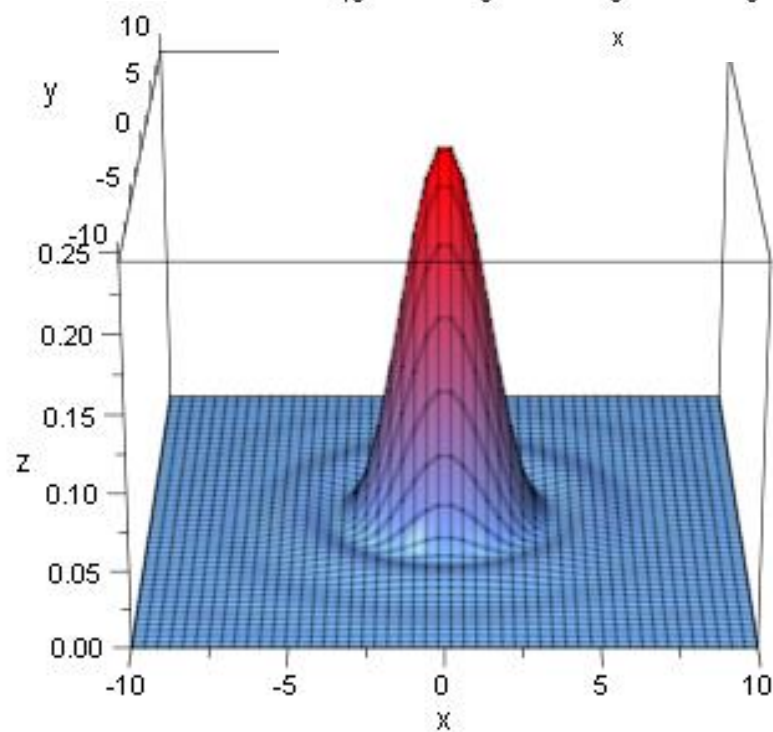
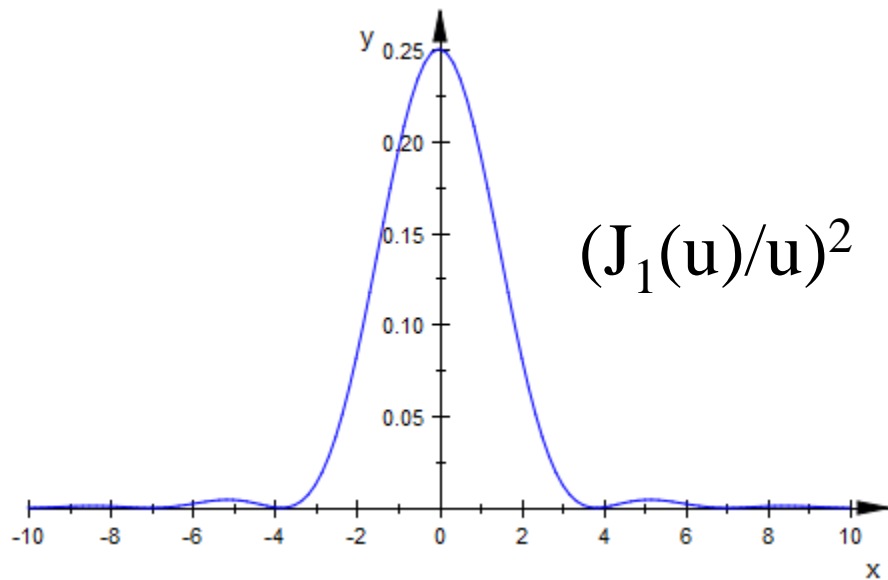
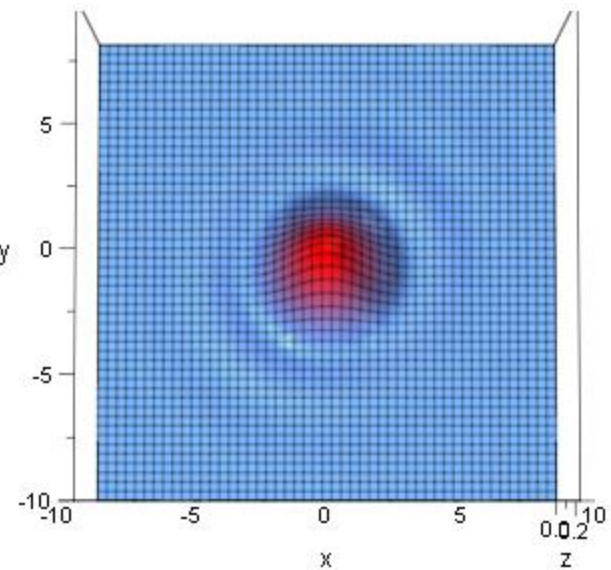
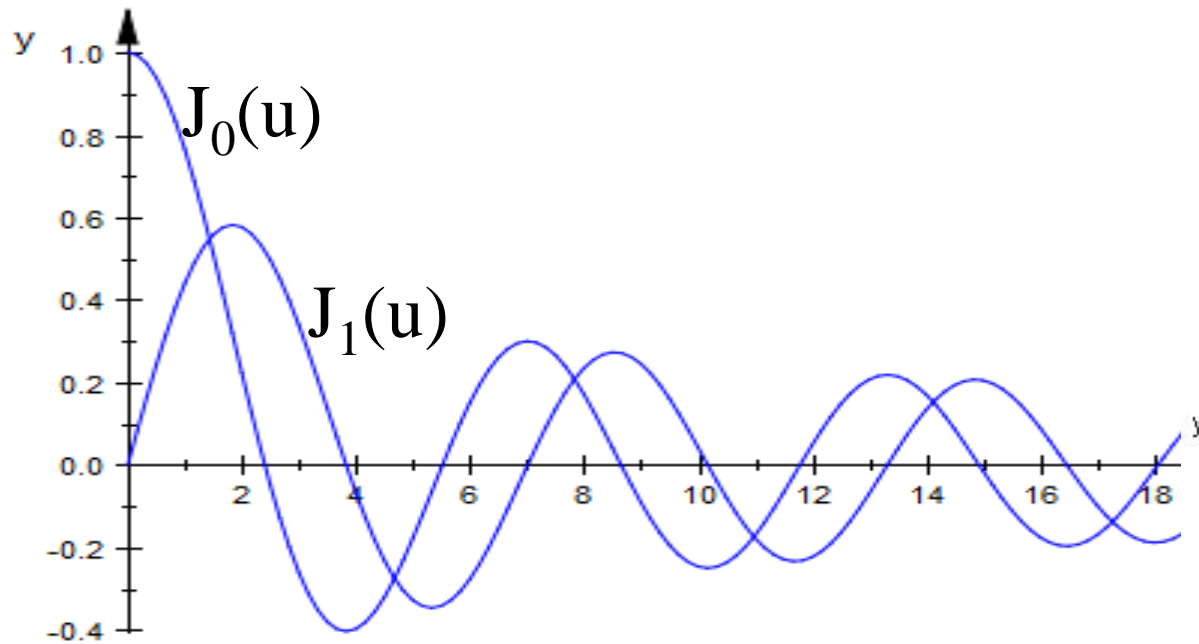
$$E = \frac{E_A e^{i(\omega t - kR)}}{R} 2\pi \int_0^a J_0\left(\frac{k\rho q}{R}\right) \rho d\rho$$

$$\frac{d}{du} [u^m J_m(u)] = u^m J_{m-1}(u) \quad \rightarrow \quad \frac{d}{du} [u^1 J_1(u)] = u^1 J_0(u)$$

$$\int_0^u u' J_0(u') du' = u J_1(u)$$

$$E(t) = \frac{E_A e^{i(\omega t - kR)}}{R} 2\pi a^2 \left(\frac{R}{kaq}\right) J_1\left(\frac{kaq}{R}\right)$$

$$I = \frac{2E_A^2 A^2}{R^2} \left[\frac{J_1\left(\frac{kaq}{R}\right)}{kaq/R} \right]^2$$

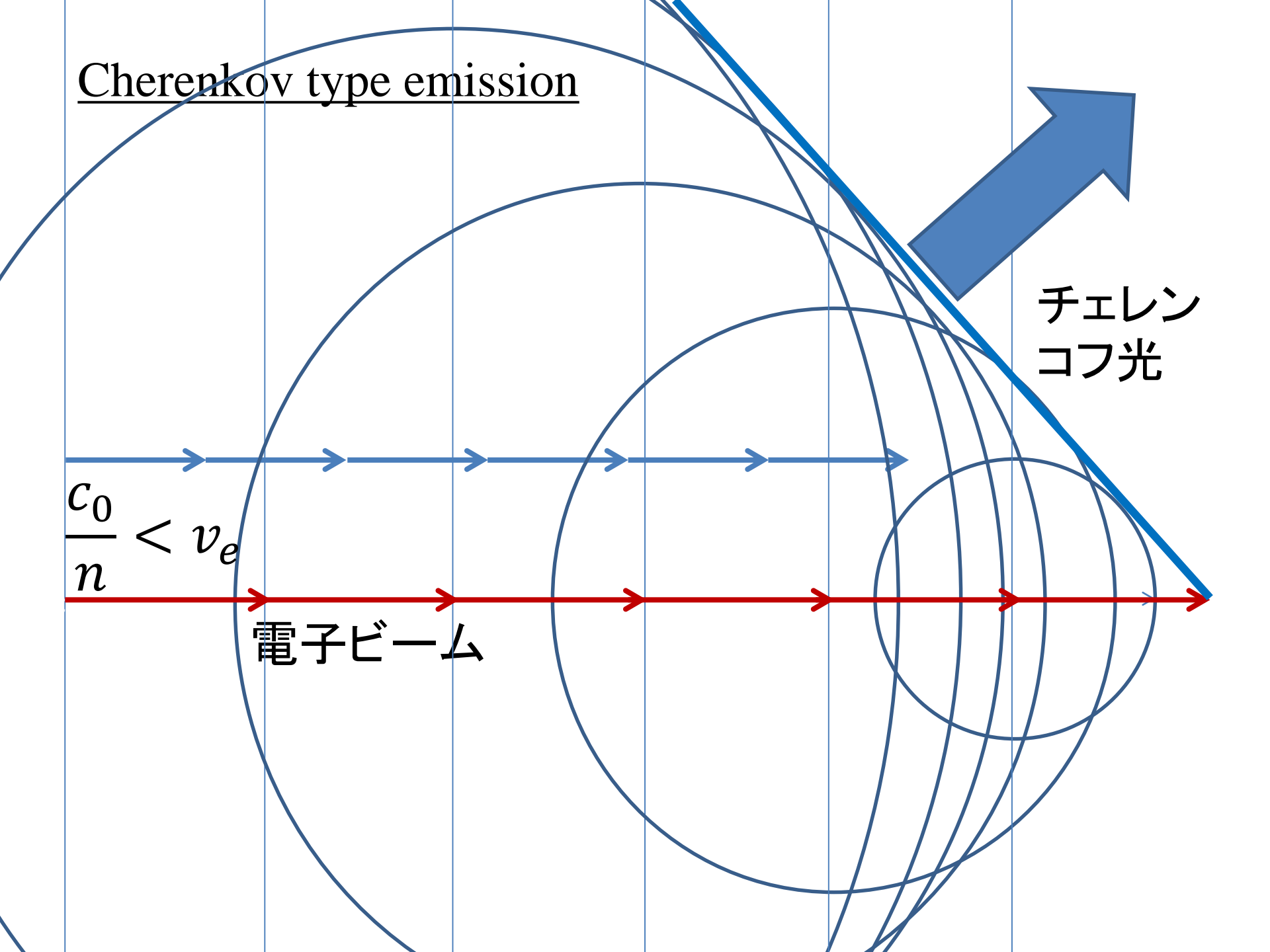


Cherenkov type emission

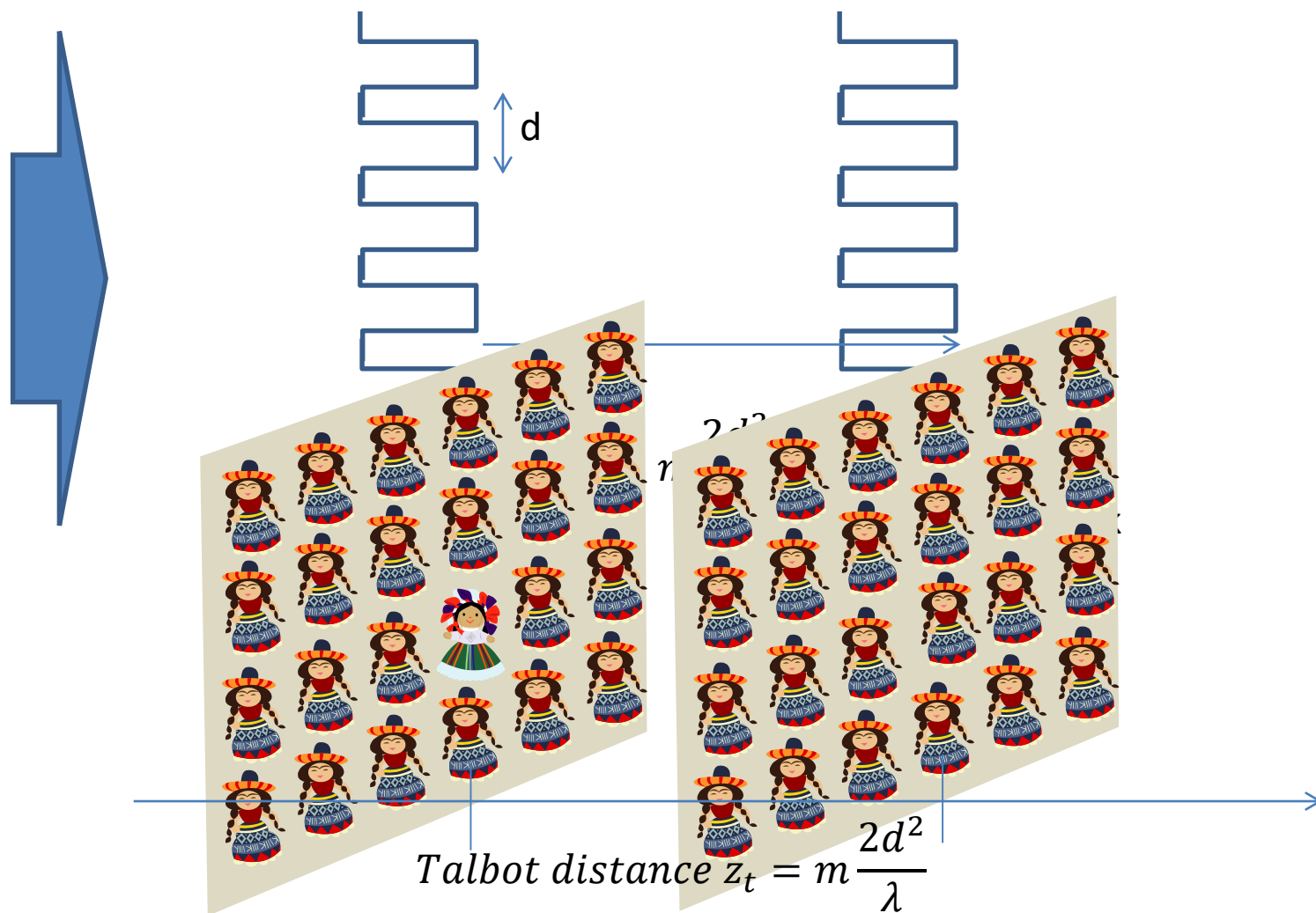
$$\frac{c_0}{n} < v_e$$

電子ビーム

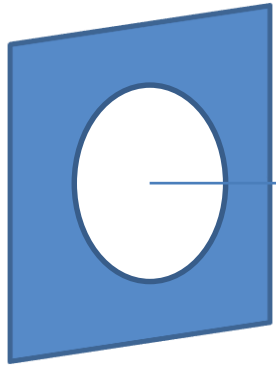
チェレンコフ光



Talbot効果 (回折で不思議な)

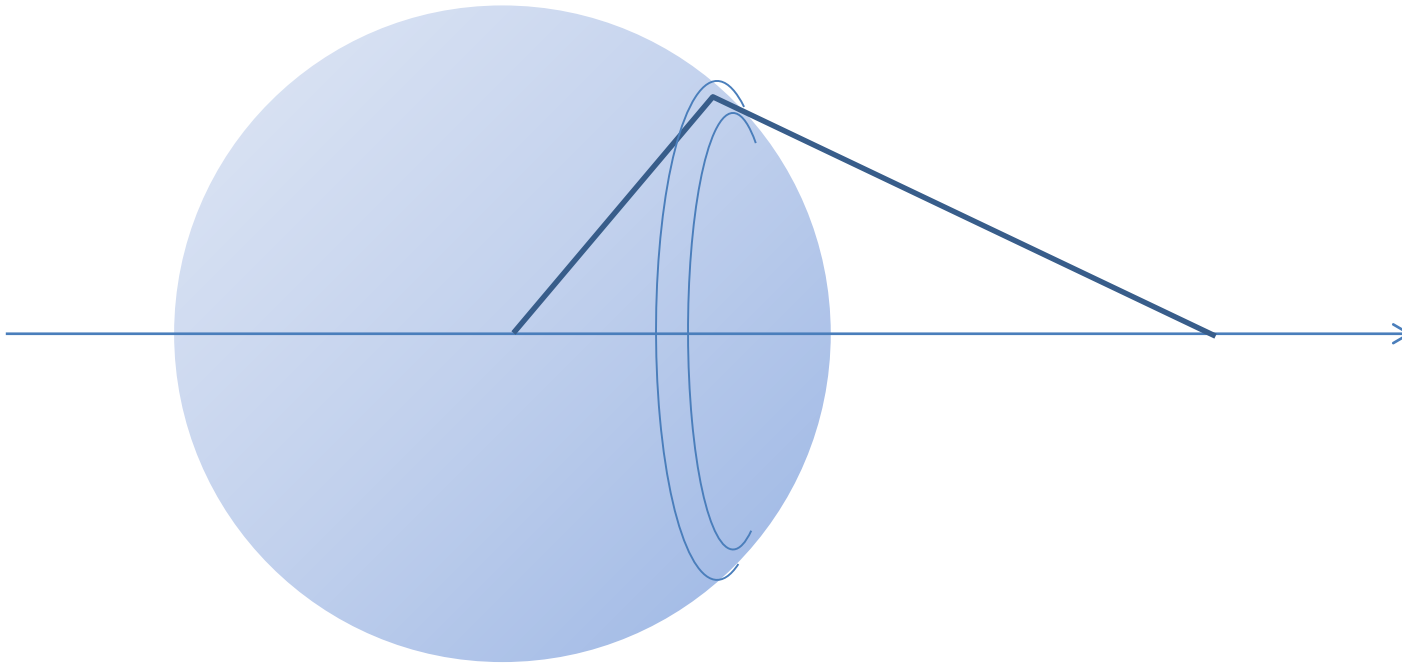


Fresnel Diffraction vs. Fraunhofer diffraction

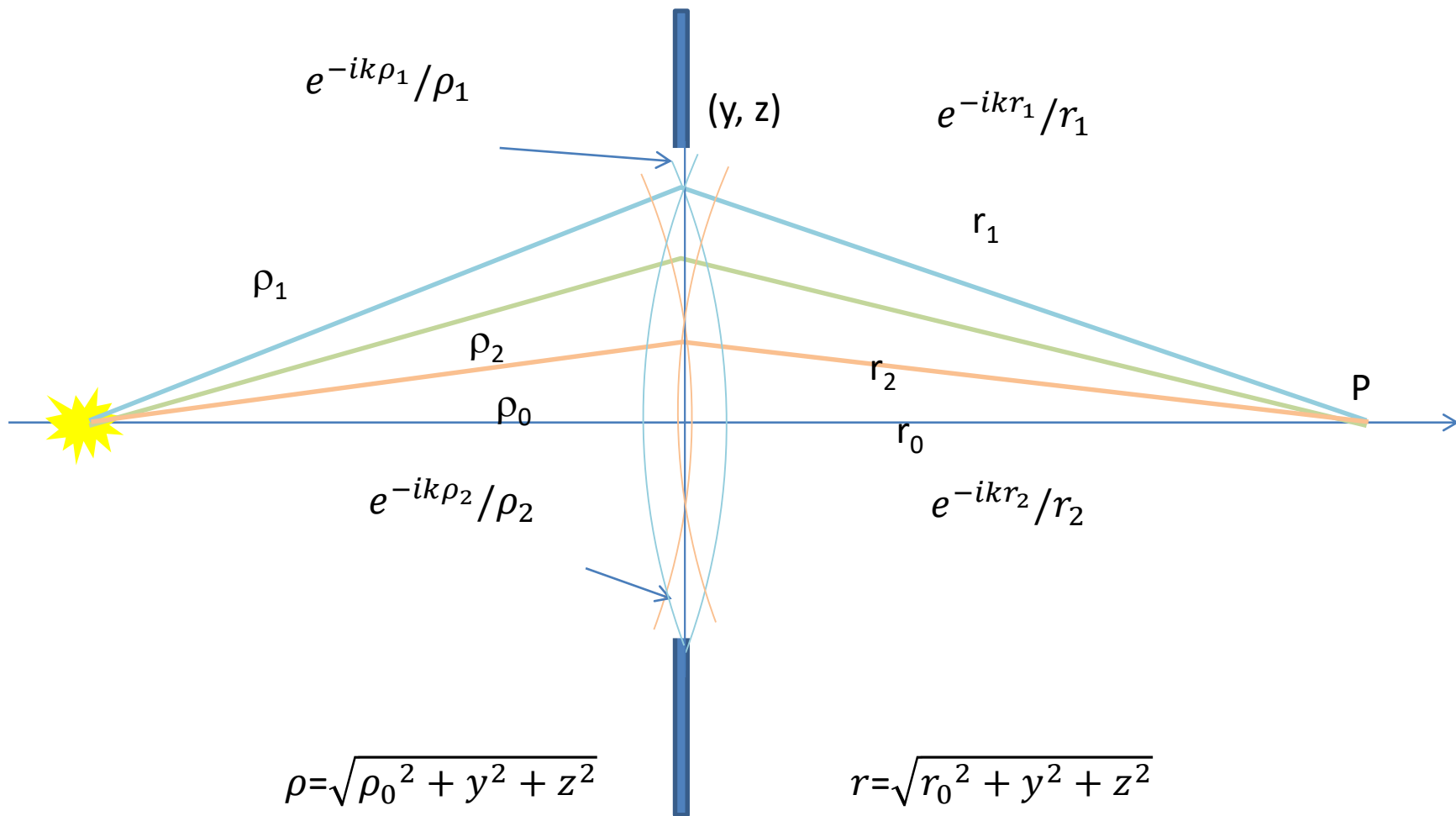


$$y, z \ll R$$

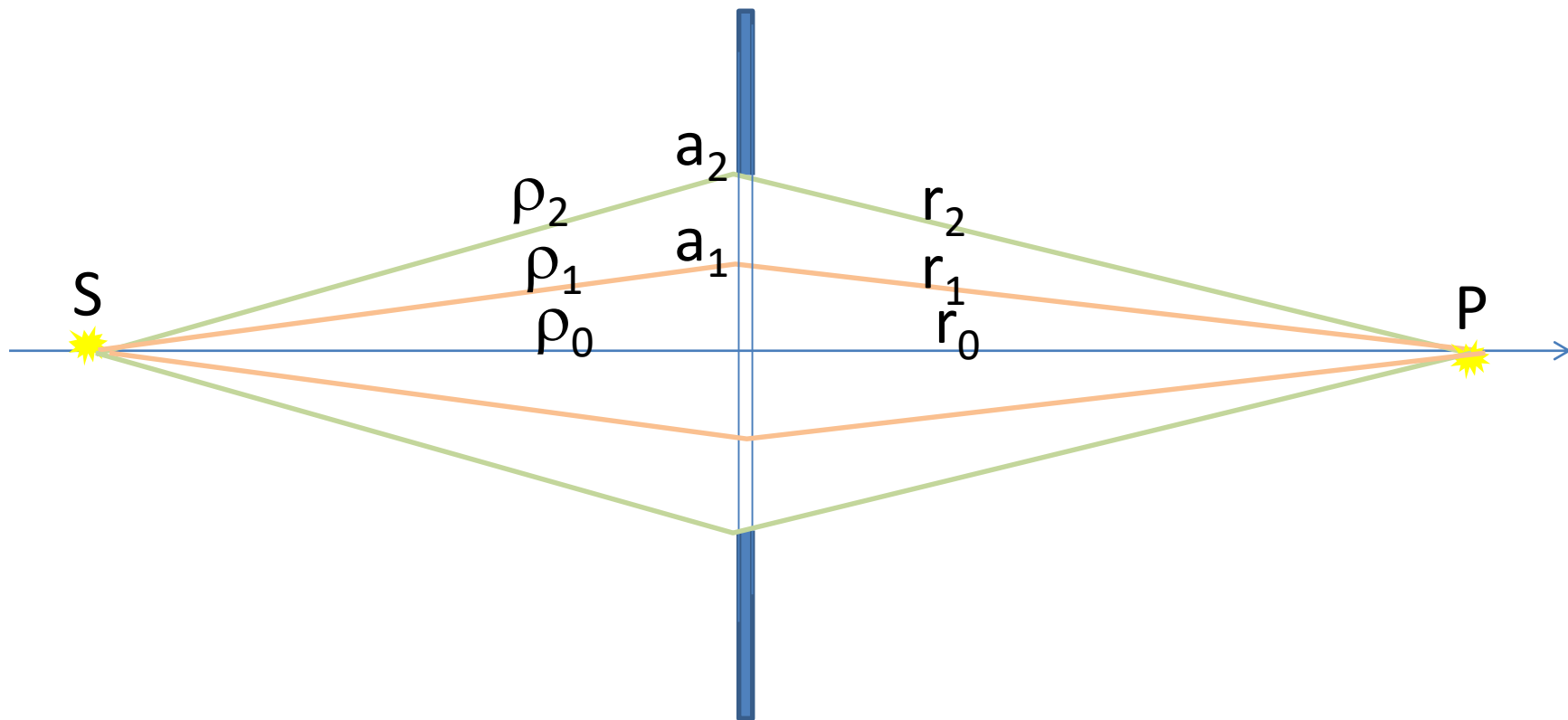
Far field
Fraunhofer



Near field
Fresnel



全部の光学距離は $\rho + r \approx \rho_0 + r_0 + (y^2 + z^2) \frac{\rho_0 + r_0}{2\rho_0 r_0}$



$$E_p = \frac{E_0 e^{-i\omega t}}{\rho_0 r_0 \lambda} \int_{y_1}^{y_2} \int_{z_1}^{z_2} e^{ik(\rho+r)} dy dz$$

ここで $u \equiv y \left[\frac{2(\rho_0 + r_0)}{\lambda \rho_0 r_0} \right]^{1/2}$ $v \equiv z \left[\frac{2(\rho_0 + r_0)}{\lambda \rho_0 r_0} \right]^{1/2}$ とすると

$$E_p = \frac{E_0 e^{-i\omega t} e^{ik(\rho_0+r_0)}}{2(\rho_0 + r_0)} \int_{u_1}^{u_2} e^{i\pi u^2/2} du \int_{v_1}^{v_2} e^{i\pi v^2/2} dv$$

ここで

$$F(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \quad G(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt \quad \int_0^x e^{i\pi t^2/2} dt = F(x) + iG(x)$$

$$E_p = \frac{E_0}{2} [F(u) + iG(u)]_{u_1}^{u_2} [F(v) + iG(v)]_{v_1}^{v_2}$$

$$I_p = \frac{I_0}{4} \{[F(u_2) - F(u_1)]^2 + [G(u_2) - G(u_1)]^2\} \times \{[F(v_2) - F(v_1)]^2 + [G(v_2) - G(v_1)]^2\} c$$

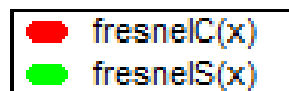
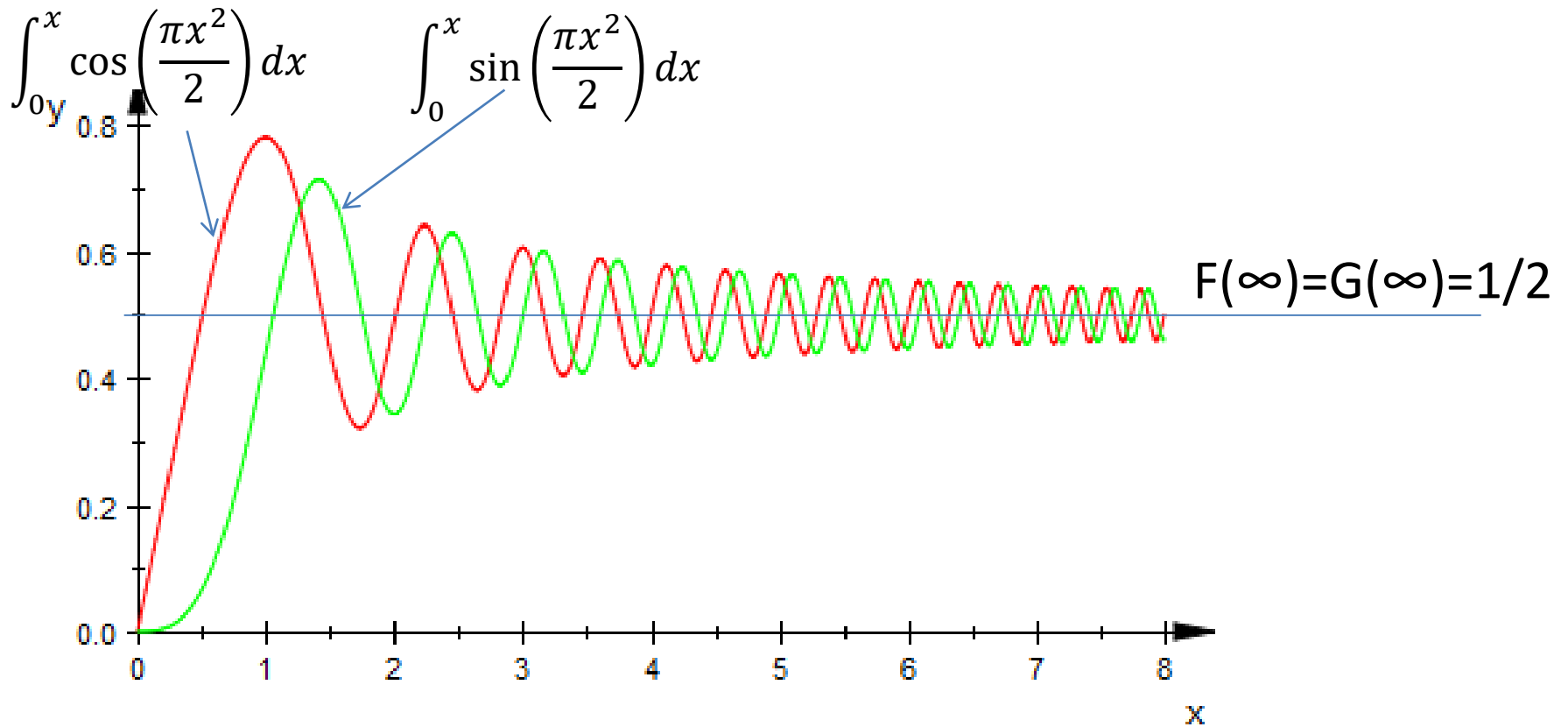
$$I_p = \frac{I_0}{4} \{ [F(u_2) - F(u_1)]^2 + [G(u_2) - G(u_1)]^2 \} \times \{ [F(v_2) - F(v_1)]^2 + [G(v_2) - G(v_1)]^2 \}$$

Fresnel integral は odd function

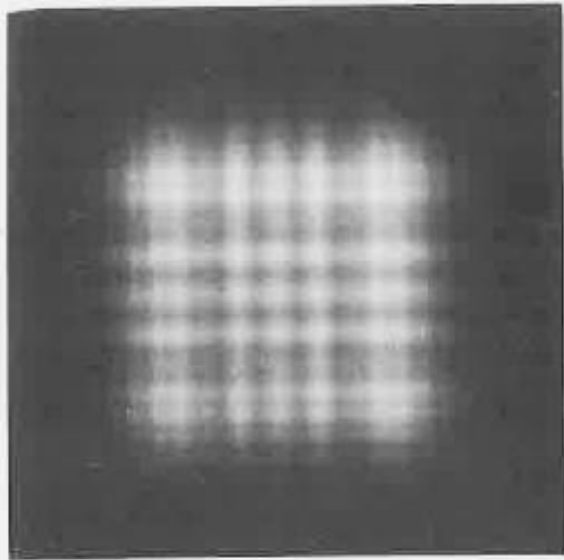
$$F(x) = -F(-x)$$

$$G(x) = -G(-x)$$

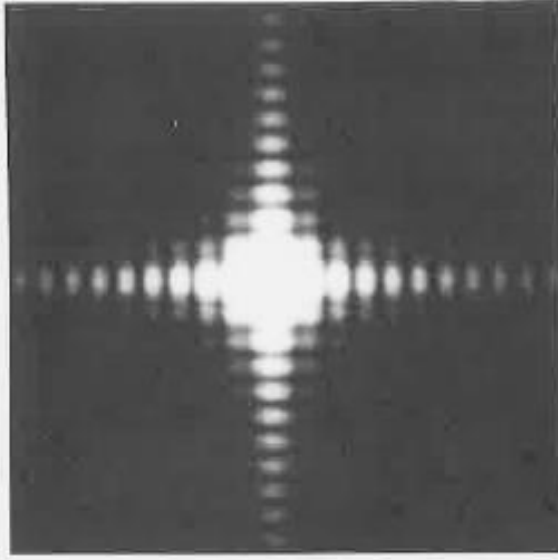
したがって、 $u_1 = -u_2 = \Delta$, $v_1 = -v_2 = \Delta$ とすると $I_p = \frac{I_0}{4} \{ [2F(\Delta)]^2 + [2G(\Delta)]^2 \}^2$



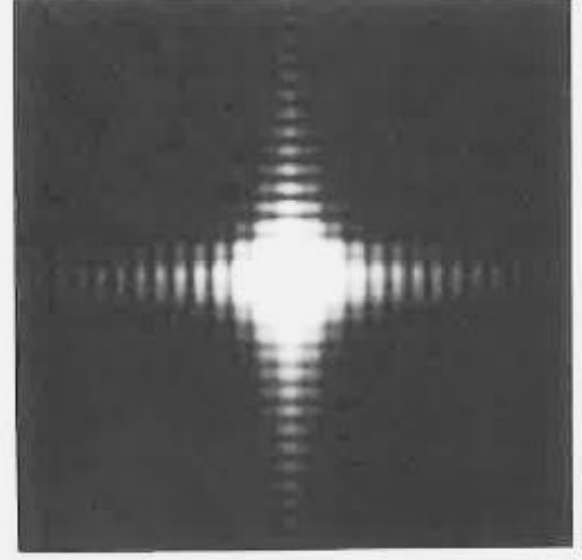
Square aperture



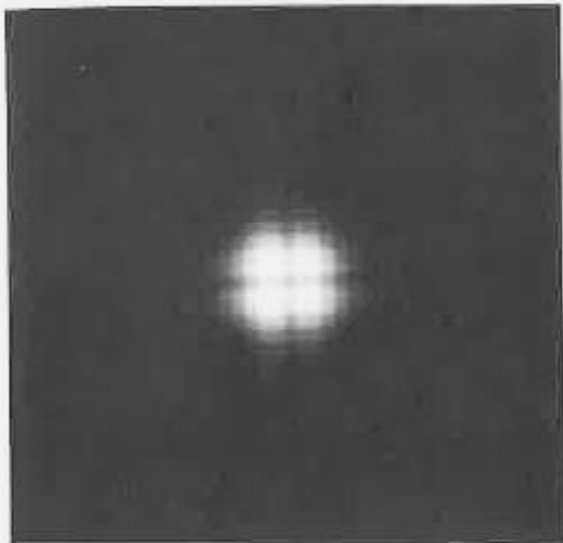
(a)



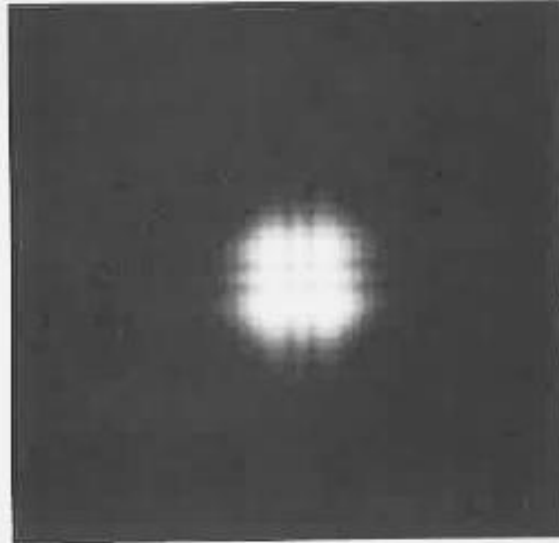
(b)



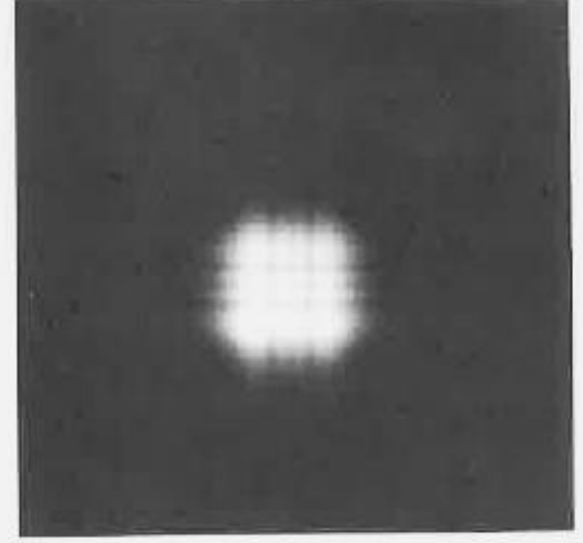
(c)



(d)

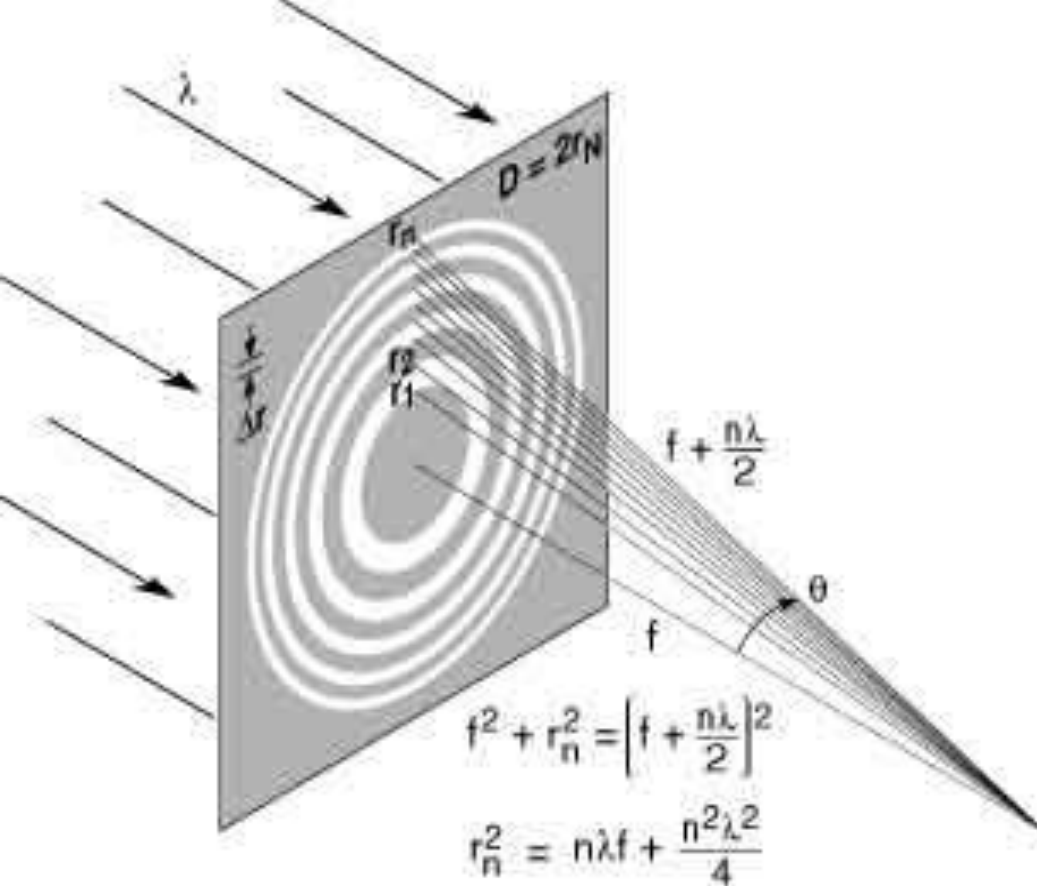


(e)

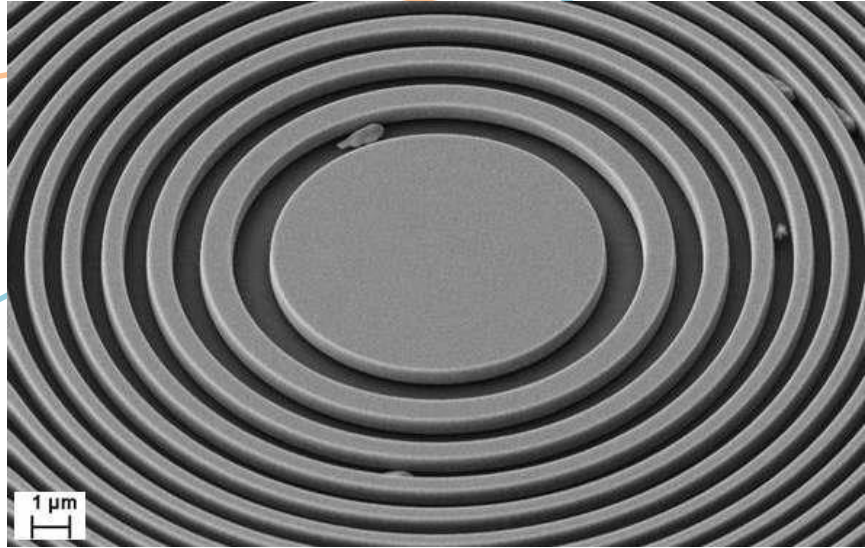
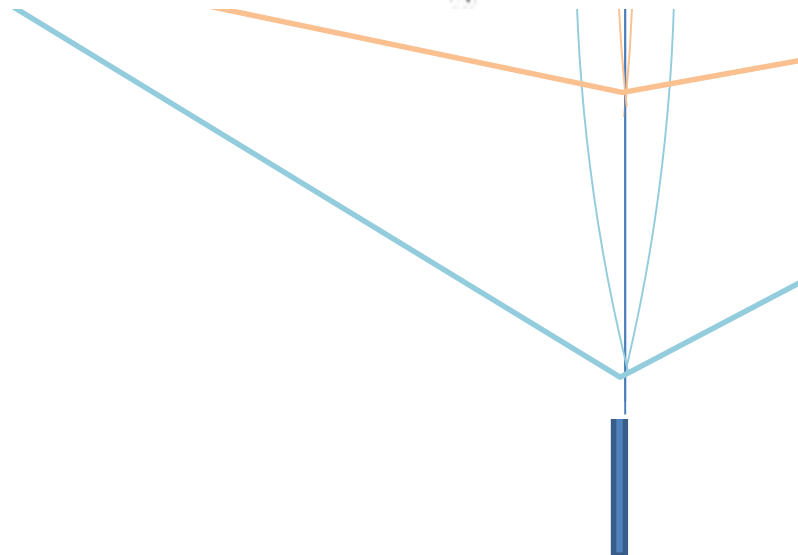
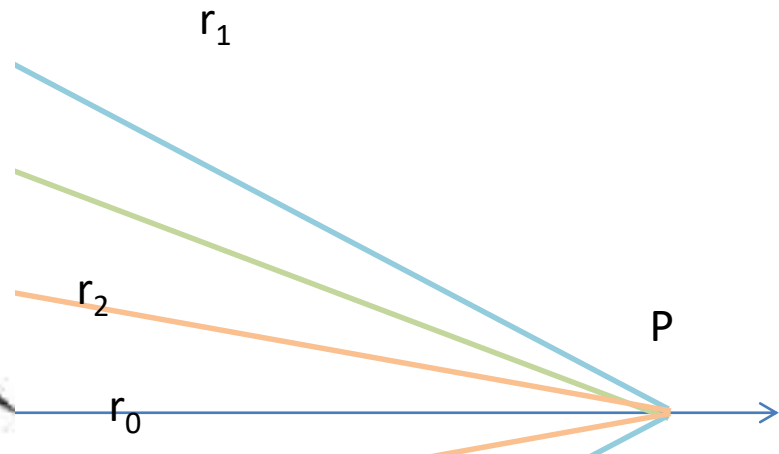


(f)

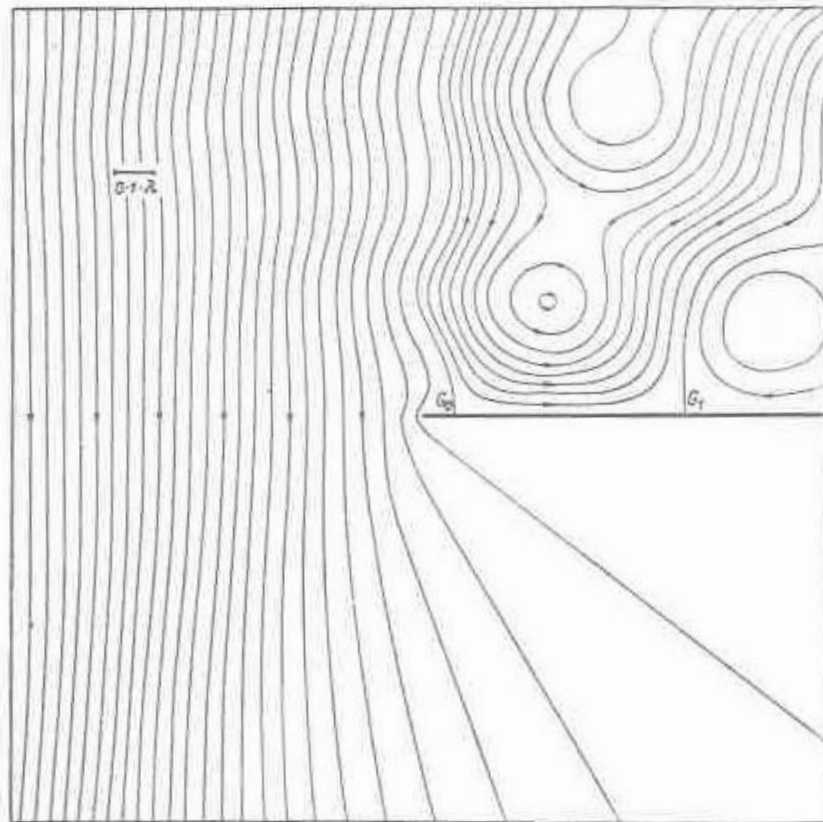
e Plate



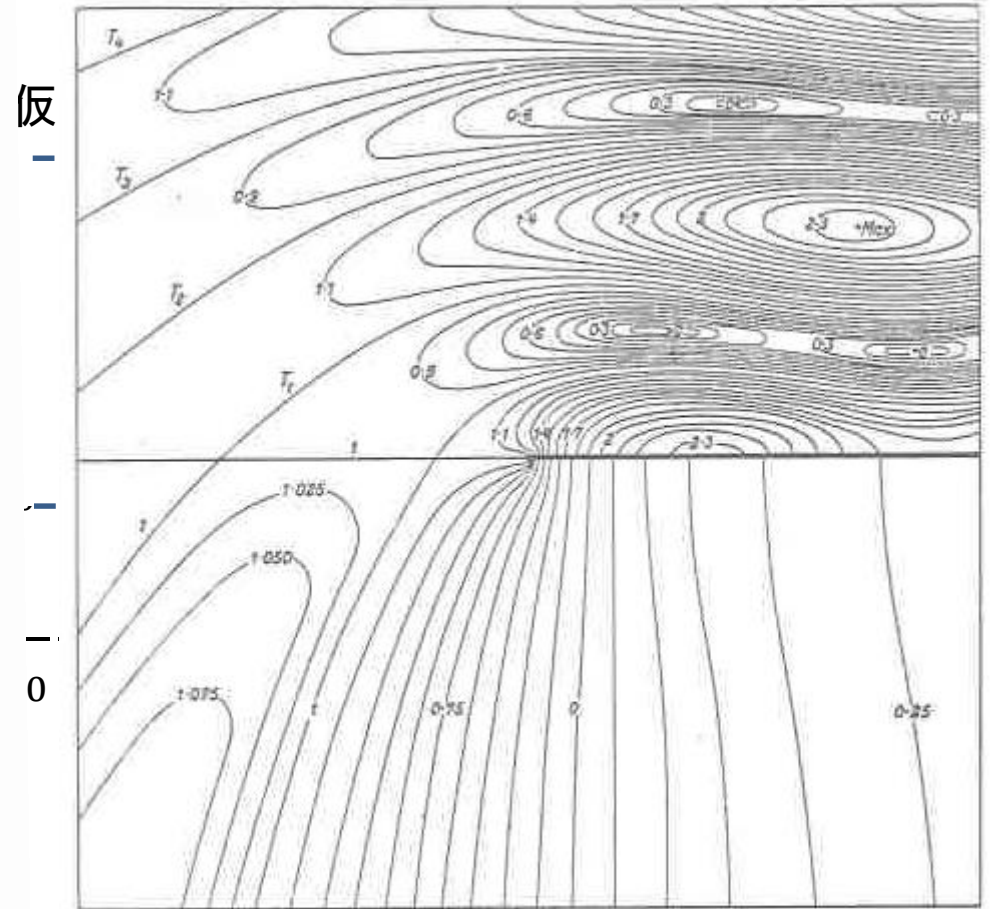
$\rho + r = 2\pi m$ を通し
 $\rho + r = 2\pi(m+1)$ をブロックする



波のモデルを使った回折理論



Line of average energy flow



Amplitude contours of Hz