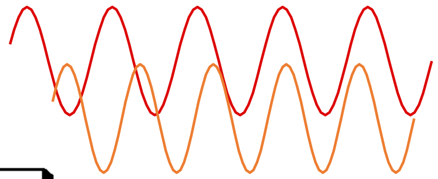
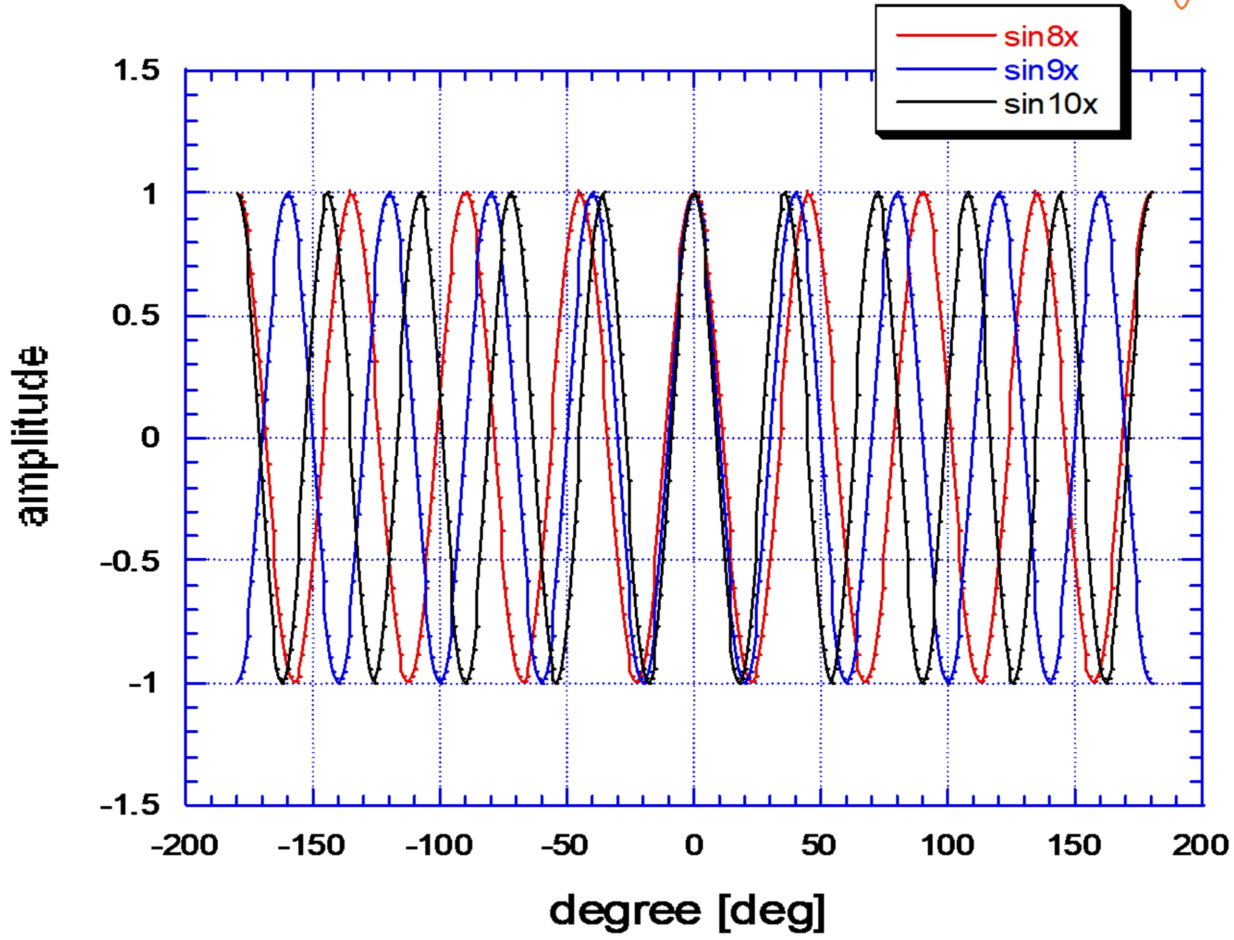
電磁波工学 XII

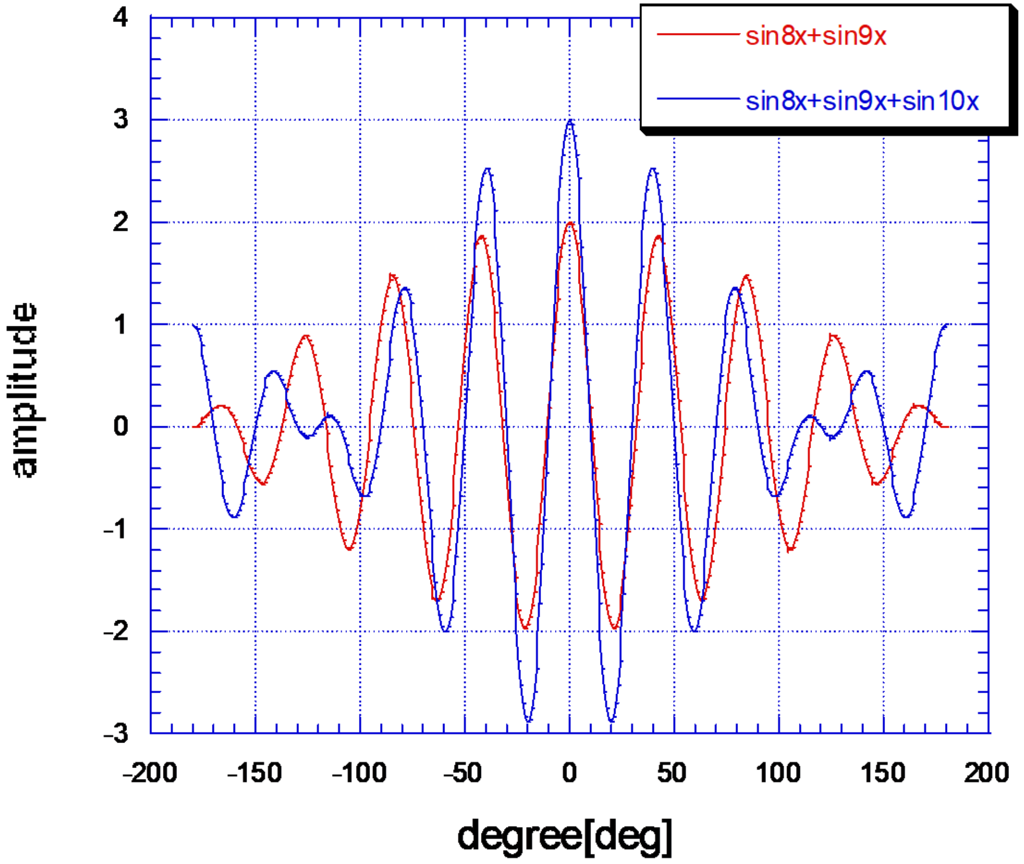
フーリエ変換、フーリエ限界、Time Domain Spectroscopy

米田仁紀

異なる周波数の波の重ね合わせ

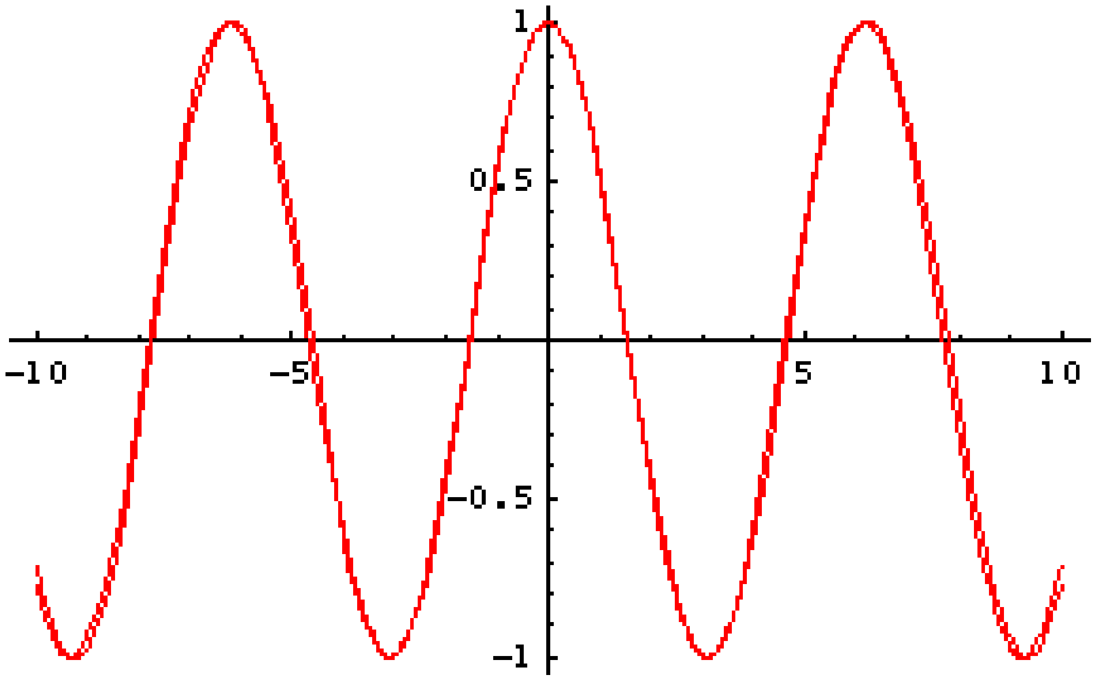


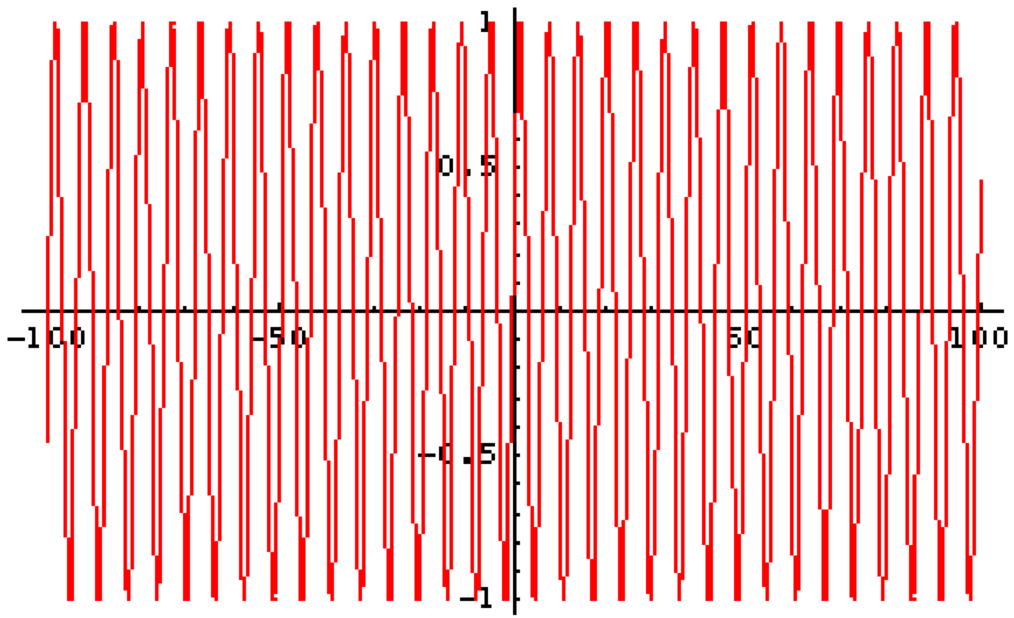


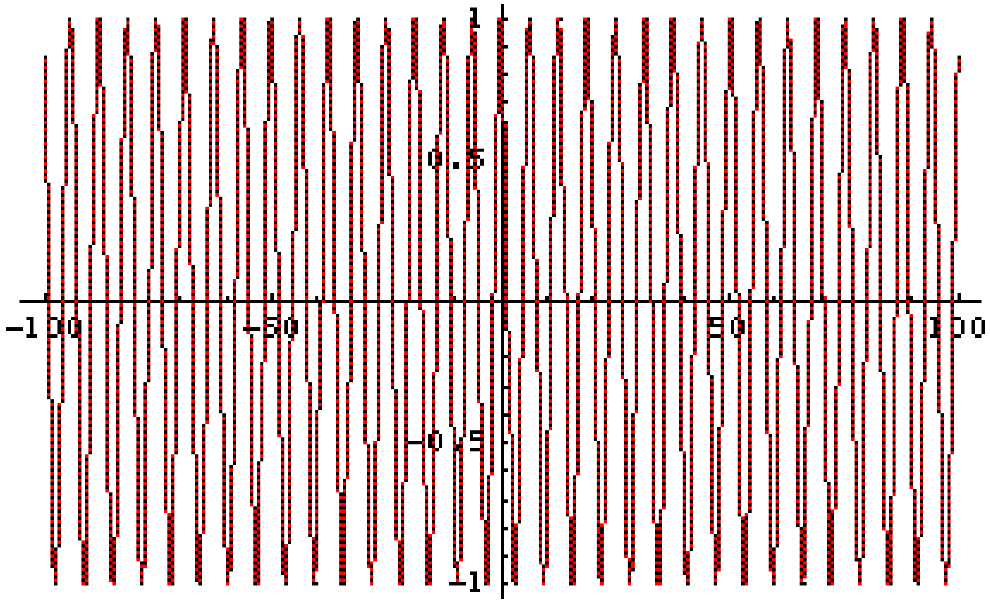


さらに多数の波の重ね合わせ

短いパルスが作られる







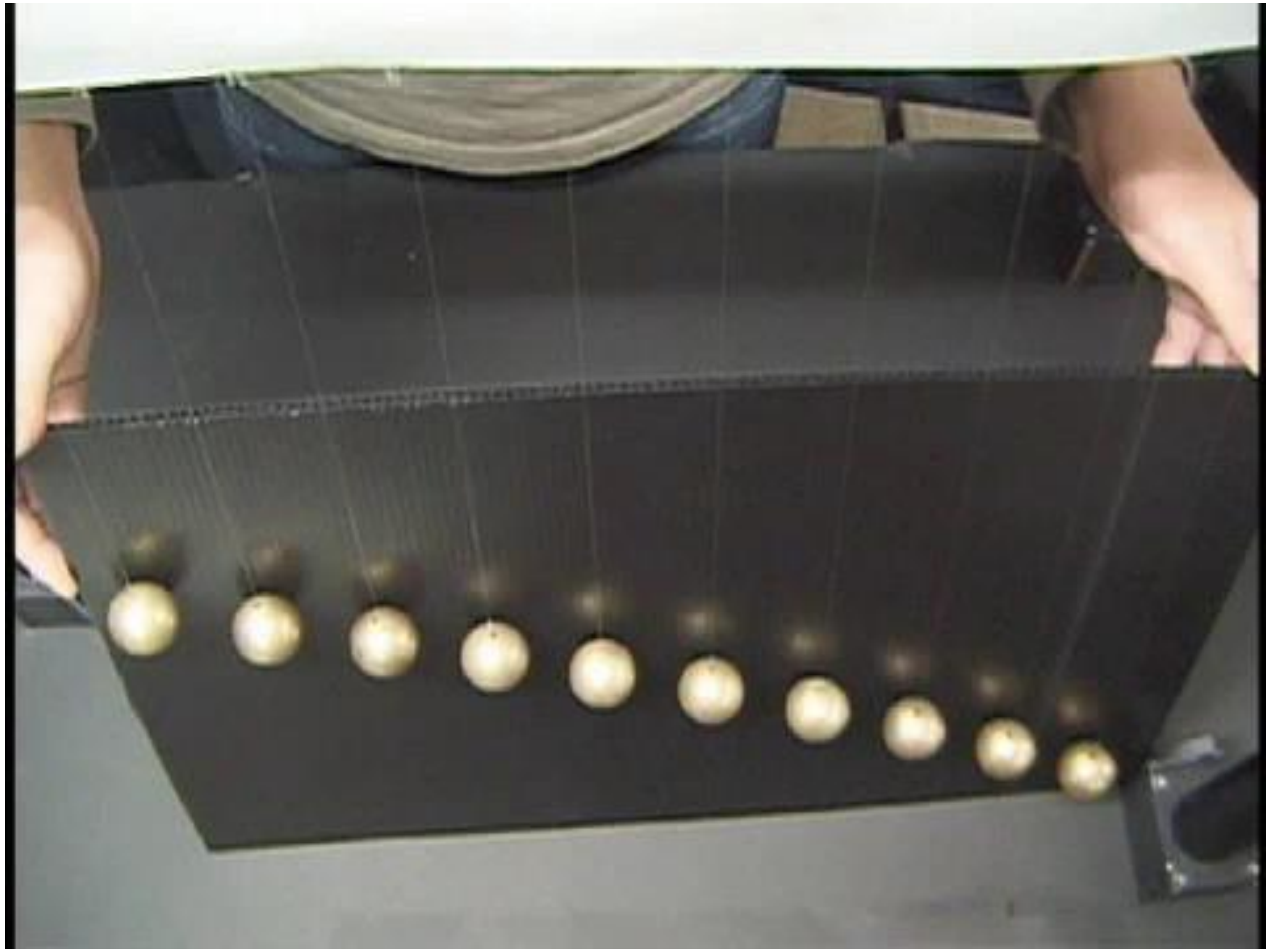
短いパルス光を作る

時間方向のモードの重ね合わせ

$$E_{total}(t) = \sum_{l=-N}^{N} E_{0l} \exp[i(\omega + l\Delta\omega)t + \alpha_l]$$

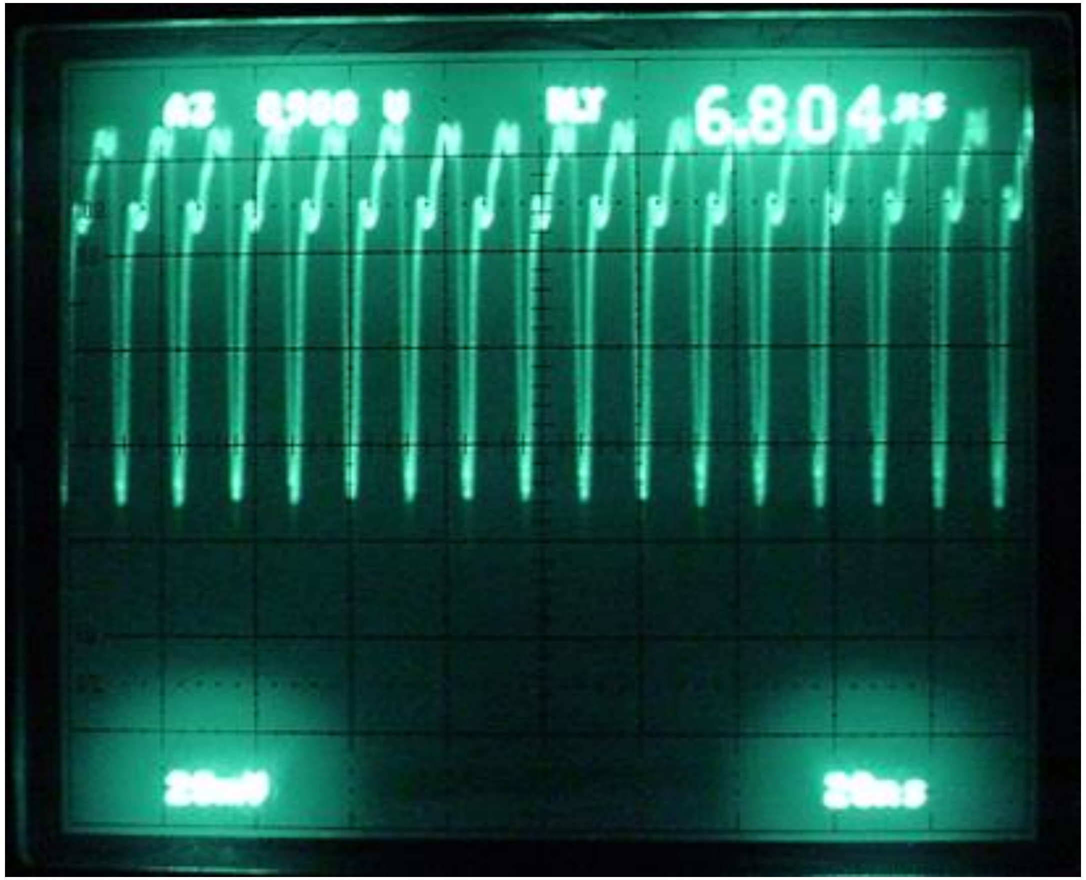
$$E_{total}(t) = E_0 \frac{\sin[(2N+1)(\Delta\omega t + \alpha)/2]}{\sin[(\Delta\omega t + \alpha)/2]} \exp(i\omega t)$$

初期位相がそろっていることに注意



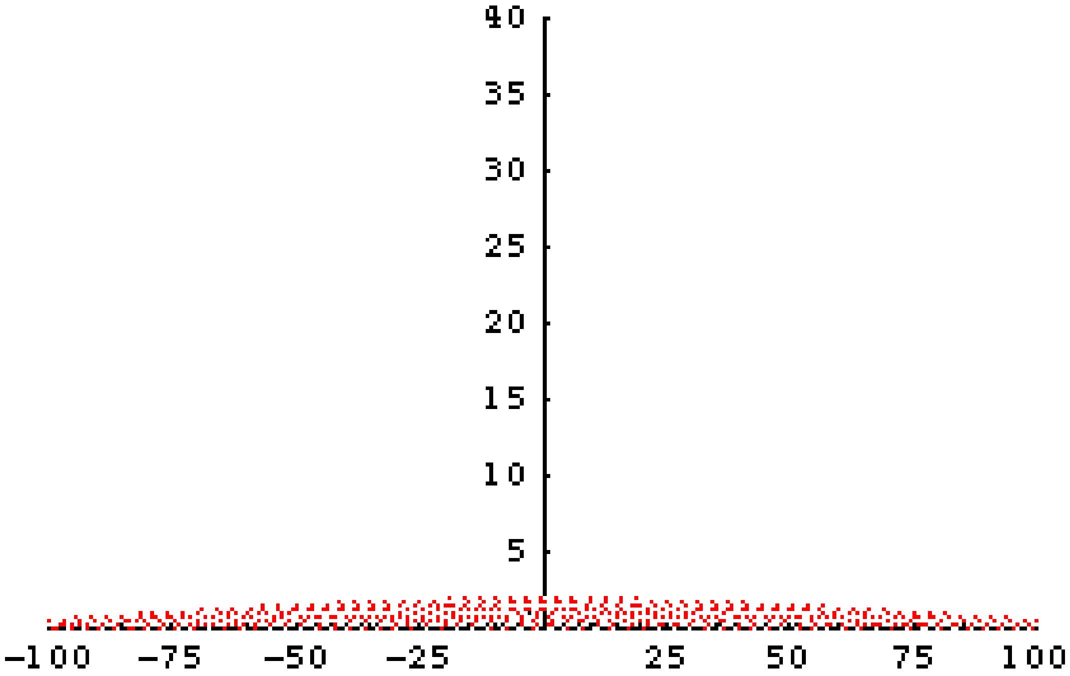
おなじ平均出力なら、短パルス化＝＞高出力化

平均1W　　ピーク強度は10kW

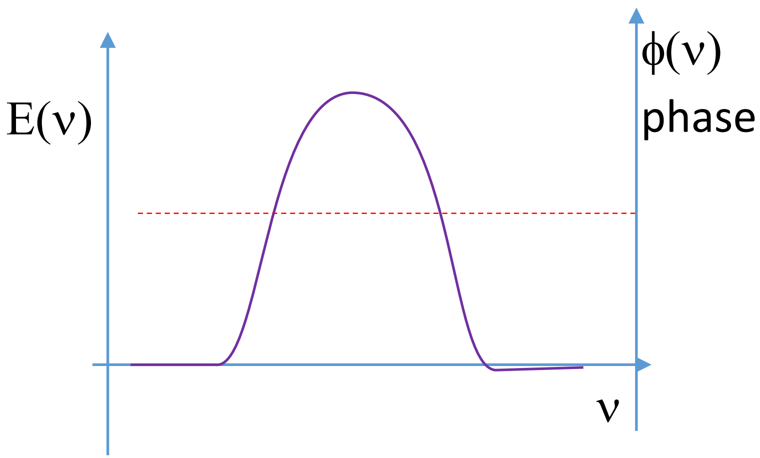


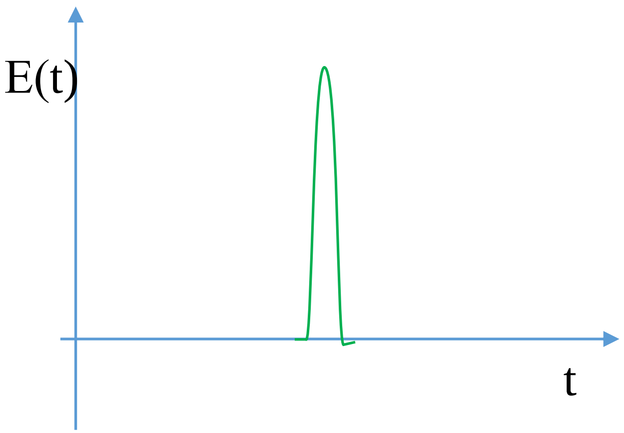
パルスが短縮される

＝＞ピークが上がる。



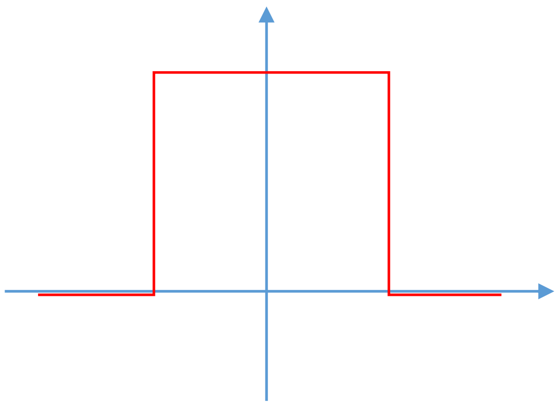
フーリエ変換





$$\int_0^{\infty} \tilde{E}(\omega)e^{i\omega t}d\omega = E(t)$$

例：2τ幅の矩形波形



$$f(t) = \begin{cases} 0 & t < -\tau, \quad t > \tau \\ 1 & -\tau \le t \le \tau \end{cases}$$

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = \frac{1}{2\pi}\int_{-\tau}^{\tau} e^{-i\omega t}dt$$

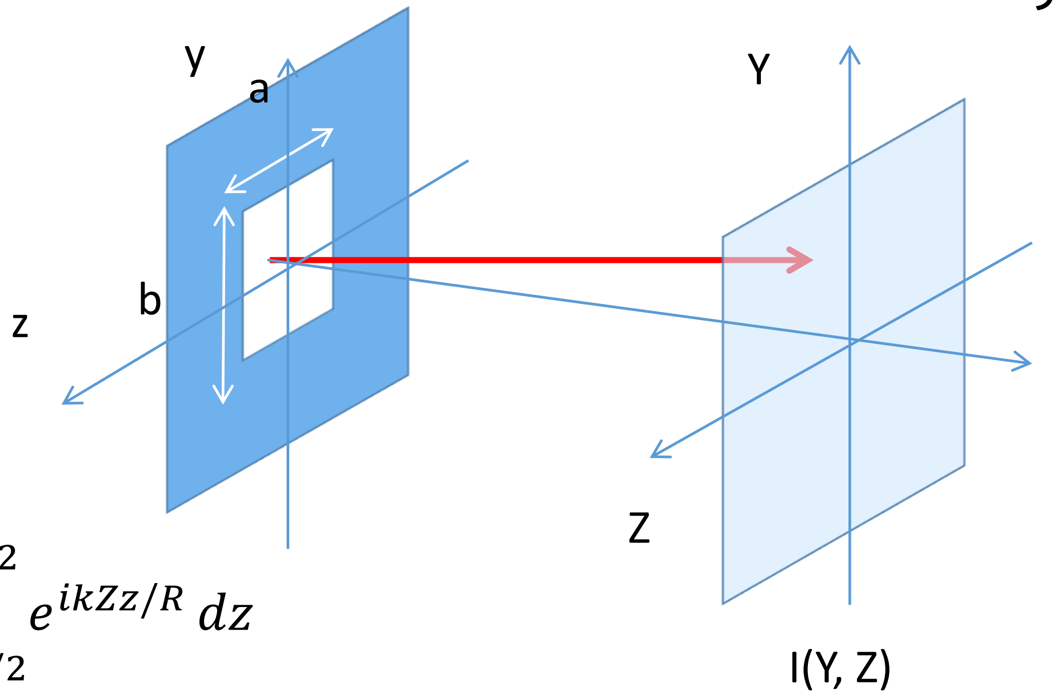
$$= \frac{1}{2\pi}\left[\frac{1}{-i\omega}e^{-i\omega t}\right]_{-\tau}^{\tau} = \frac{\tau}{\pi}\frac{\sin\omega\tau}{\omega\tau}$$

$$F(\omega) \sim \mathrm{sinc}(\omega\tau)$$

Fraunhofer diffraction

Rectangular Aperture

$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \iint e^{ik(Yy+Zz)/R} ds$$



$$E = \frac{E_A e^{i(\omega t - kR)}}{R} \int_{-b/2}^{b/2} e^{ikYy/R} dy \int_{-a/2}^{a/2} e^{ikZz/R} dz$$

ここで、$\beta = \frac{kbY}{2R}$, $\alpha = kaZ/2R$とすると

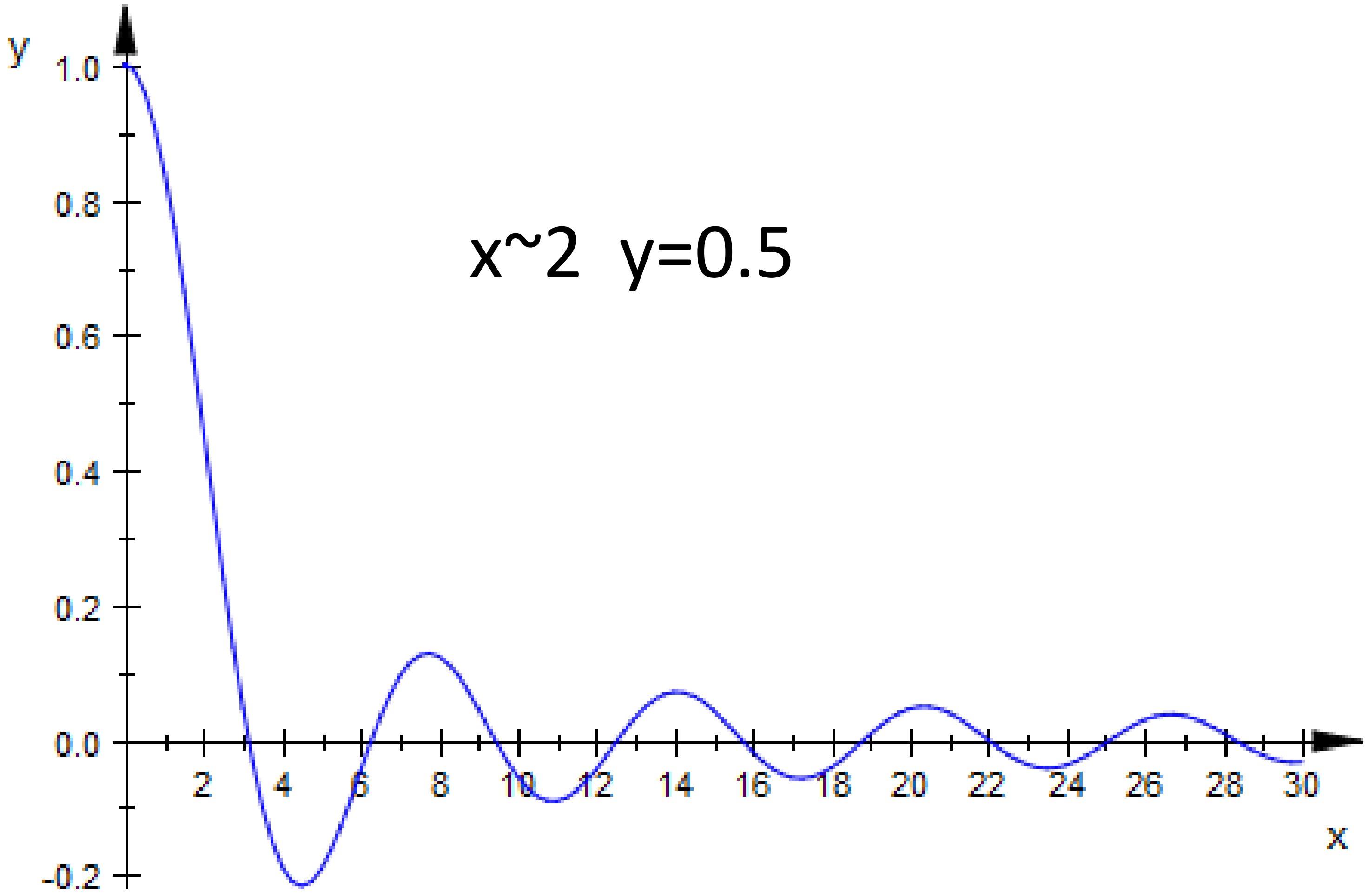
$$\int_{-b/2}^{b/2} e^{ikYy/R} d\,y = b\left(\frac{e^{i\beta} - e^{-i\beta}}{2i\beta}\right) = b\left(\frac{\sin\beta}{\beta}\right)$$

$$\mathrm{E} = \frac{AE_A e^{i(\omega t - kR)}}{R}\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\sin\beta}{\beta}\right)$$

sinc関数！

$$I(Y,Z) = I(0)\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin\beta}{\beta}\right)^2$$

sinc(x)



x~2 y=0.5

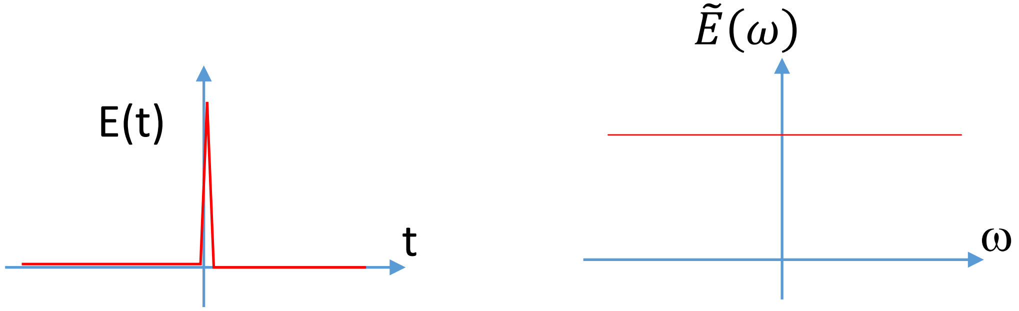
δτ=2τ のパルス幅

δωδτ ~ 4

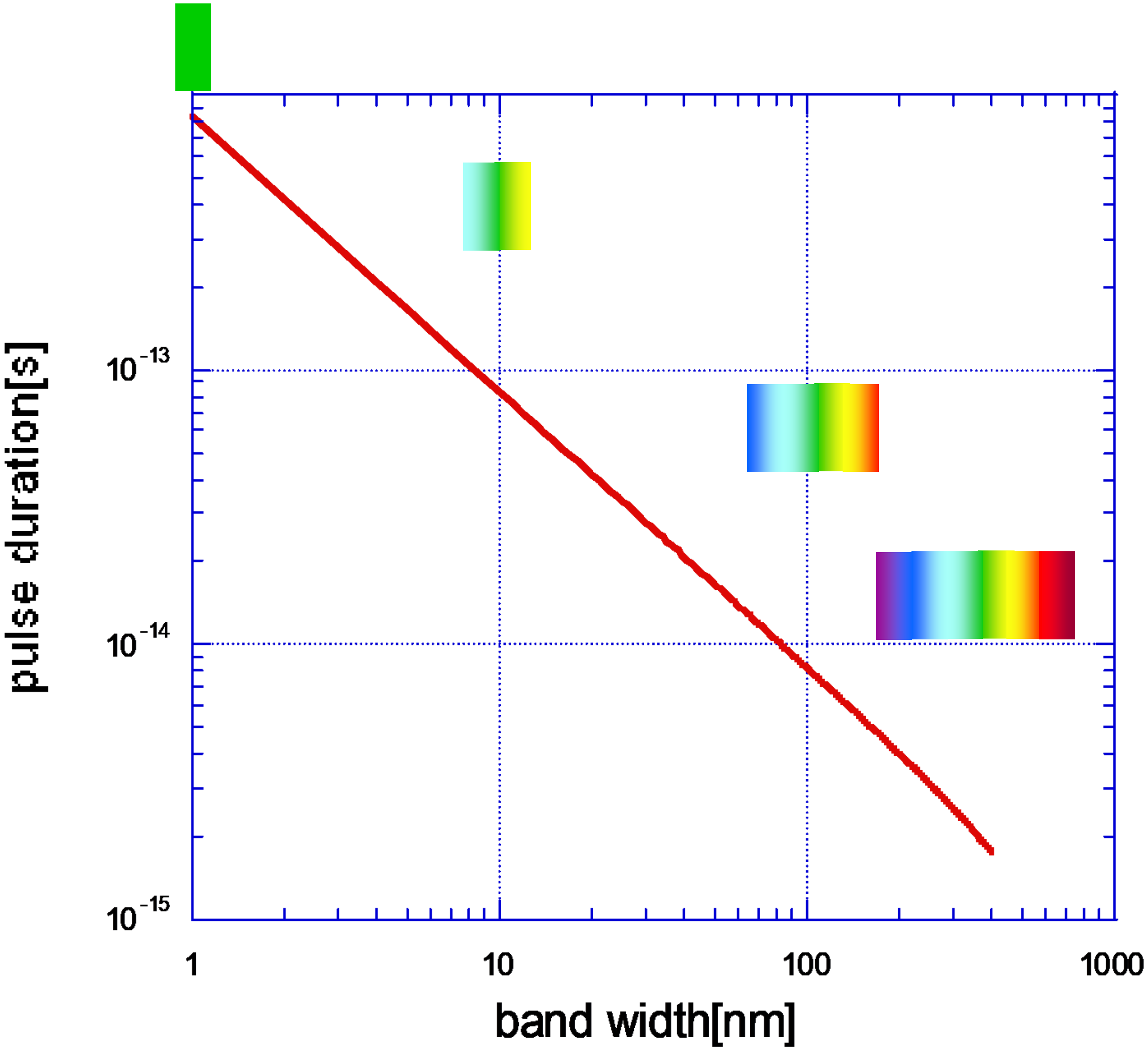
δω ~2/δτ

フーリエ限界パルス

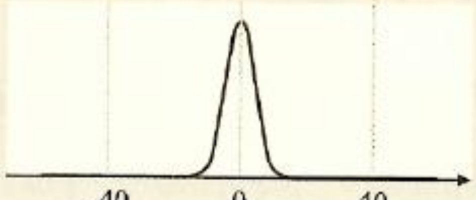
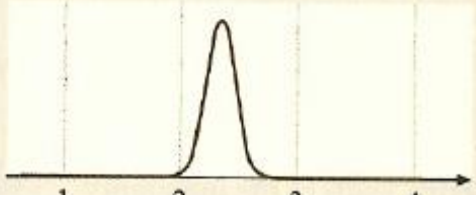
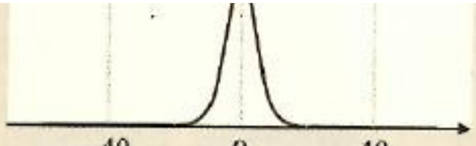
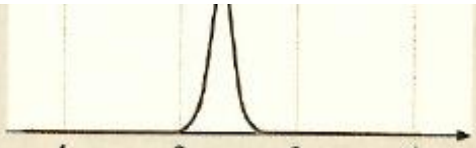
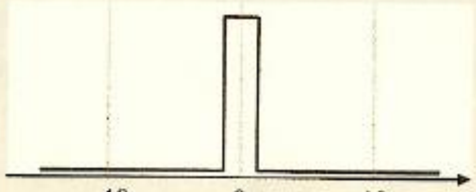
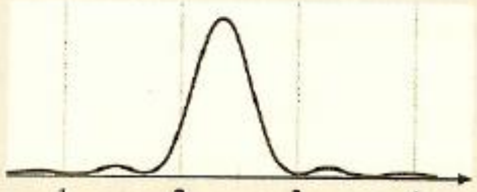
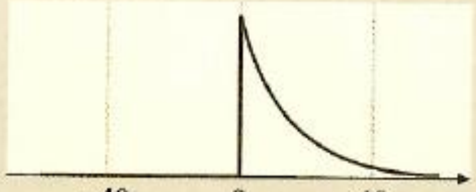
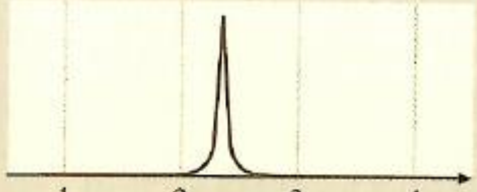
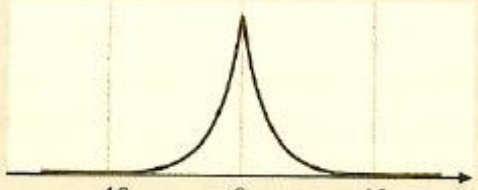
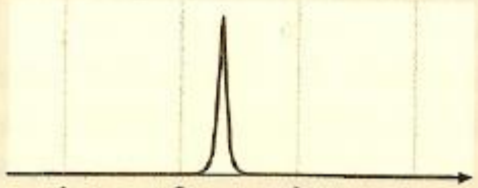
$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt$$



$$\Delta\nu \times \Delta t \geq 1$$

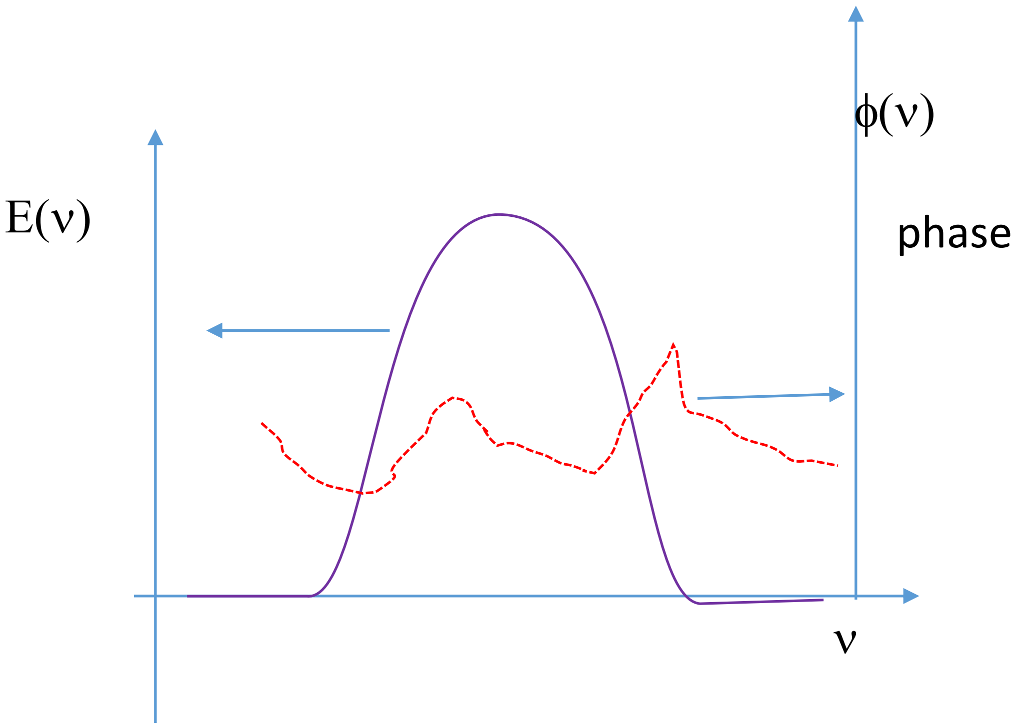


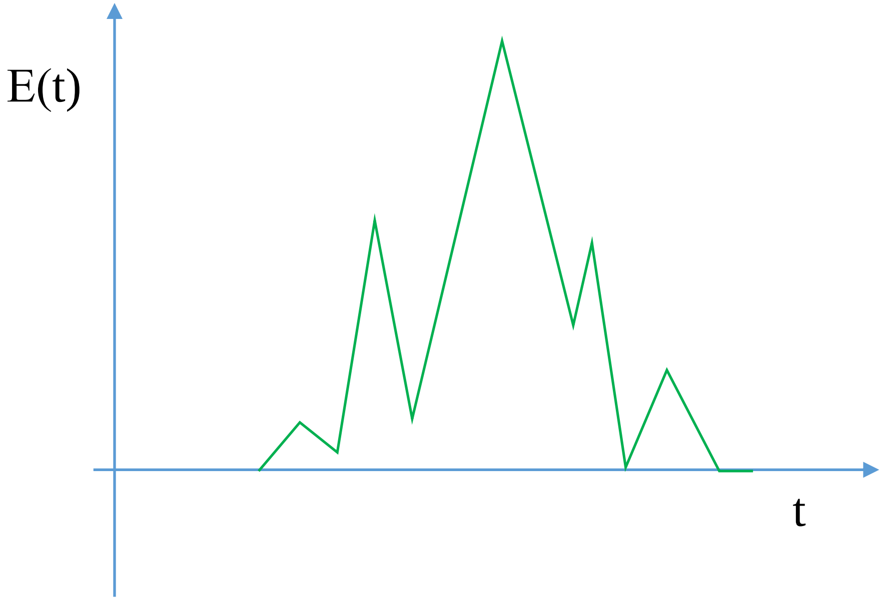
Time bandwidth product

Shape	$I(t)$	$I(\omega)$	$\Delta\nu\cdot\Delta t$	$\Delta t_{\mathrm{intAC}}/\Delta t$
Gaussian			0.441	1.414
Hyperbolic sechant			0.315	1.543
Square			0.886	1.000
Single sided exponential			0.110	2.000
Symmetric exponential			0.142	2.421

Gaussianはフーリエ変換をしてもGaussianになる。

実際にはE(ν)は複素数　（振幅と位相）

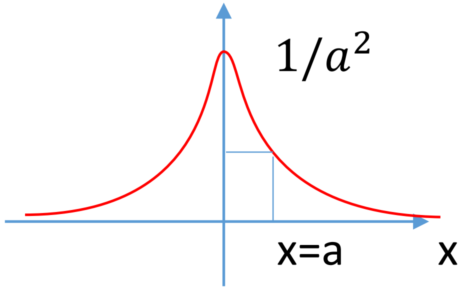


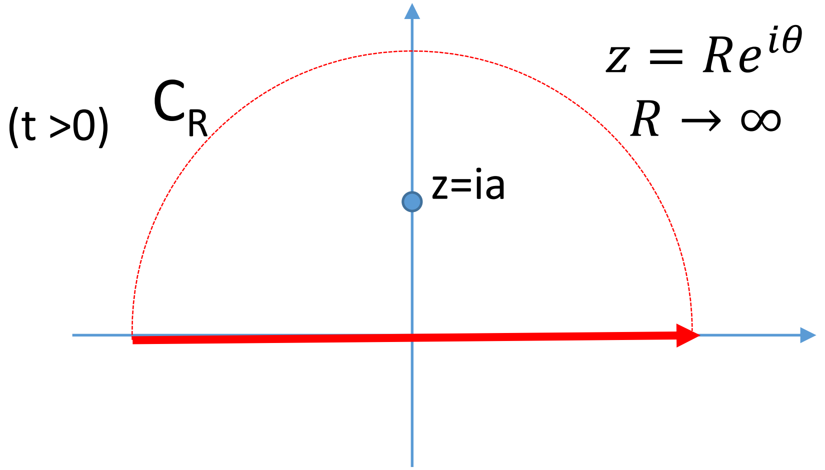


$$\int_0^{\infty} \tilde{E}(\omega) e^{i\omega t + \phi(\omega)} d\omega = E'(t)$$

Fourier型の積分の解法例

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + a^2} e^{-i\omega x} dx$$





$$\int_{C_R+(-R\sim R)} \frac{e^{-i\omega z}}{z^2 + a^2} dz = \int_{-\infty}^{\infty} \frac{1}{x^2 + a^2} e^{-i\omega x} dx + \int_{C_R} \frac{e^{-i\omega z}}{z^2 + a^2}$$

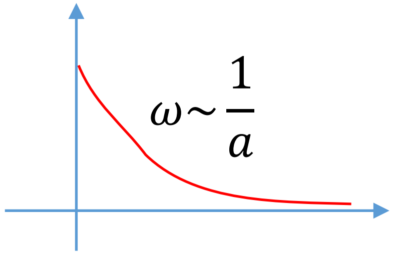
$z = \pm ia$で特異点

$$\int_{C_R+(-R\sim R)} \frac{e^{-i\omega z}}{z^2 + a^2} dz = \int_{C_R+(-R\sim R)} \frac{e^{-i\omega z}}{(z-ia)(z+ia)} dz = 2\pi i \mathrm{Res}\left(\frac{e^{-i\omega z}}{(z-ia)(z+ia)}, z = ia\right) = 2\pi i \frac{e^{a\omega}}{2ia} = \frac{\pi}{a} e^{a\omega}$$

$$dz = Rie^{i\theta} d\theta,$$

$$\int_{C_R} \frac{e^{-i\omega z}}{z^2 + a^2} dz \qquad \left|\int_{C_R} \frac{e^{-i\omega z}}{z^2 + a^2} dz\right| = \int_{C_R} \frac{R}{R^2 + a^2} d\theta \sim \frac{2\pi}{R} = 0 \qquad R \to \infty$$

$$\int_{-\infty}^{\infty} \frac{e^{-i\omega x}}{x^2 + a^2} dx = \frac{\pi}{a} e^{a\omega}$$



$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2} dx = \frac{\pi}{a} e^{-a}$$

複素関数の積分：特異点と留数

$$\frac{1}{2\pi i}\oint_C f(z)dz = \mathrm{Res}(f, z_0)$$

$z = z_0$ がf(z)とその積分範囲で唯一の特異点の時、
積分の値は2πi Res(f(z), z_0)となる。

複数の１次特異点がある場合、その積分値は

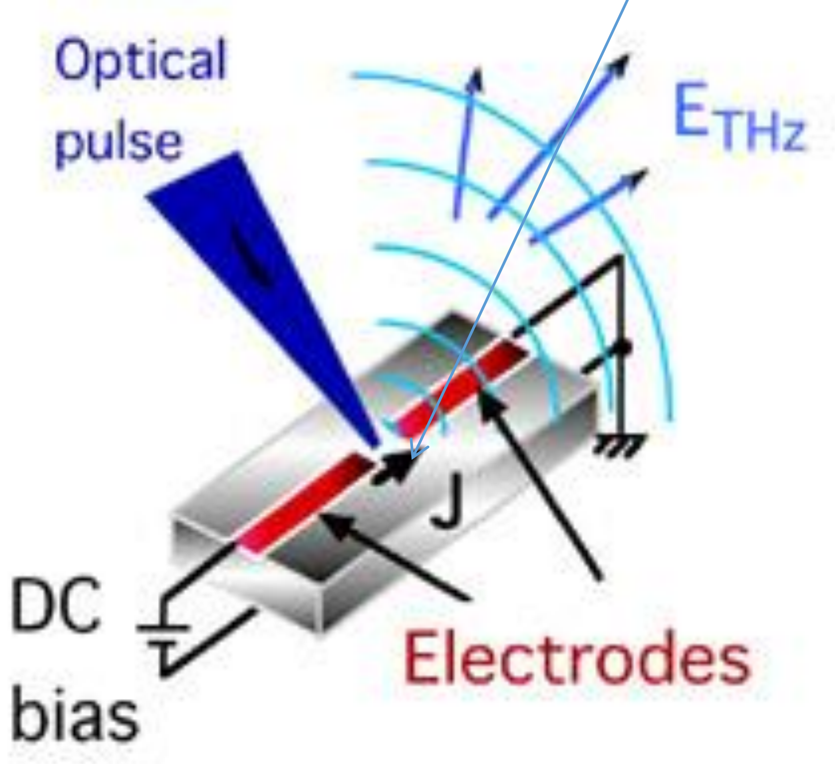
$$\oint_C f(z)\,dz = 2\pi i \sum \mathrm{Res}(f(z), z_i)$$

短電流パルス＝＞パルス幅の逆数の周波数電磁波

$$\mathbf{E}(\mathbf{r},t) = -\frac{1}{4\pi\varepsilon_0}\int_{V'}\int_{-\infty}^{t}\left(\frac{3([\mathbf{J}]\cdot\mathbf{R})\mathbf{R}}{R^5} - \frac{[\mathbf{J}]}{R^3}\right)dt'dv' + \frac{1}{4\pi\varepsilon_0}\int_{V'}\left(\frac{3([\mathbf{J}]\cdot\mathbf{R})\mathbf{R}}{R^4} - \frac{[\mathbf{J}]}{cR^2}\right)dv' + \frac{1}{4\pi\varepsilon_0}\frac{1}{c^2}\int_{V'}\frac{\left(\left[\frac{\partial}{\partial t}\mathbf{J}\right]\times\mathbf{R}\right)\times\mathbf{R}}{R^3}dv'$$

光伝導スイッチ

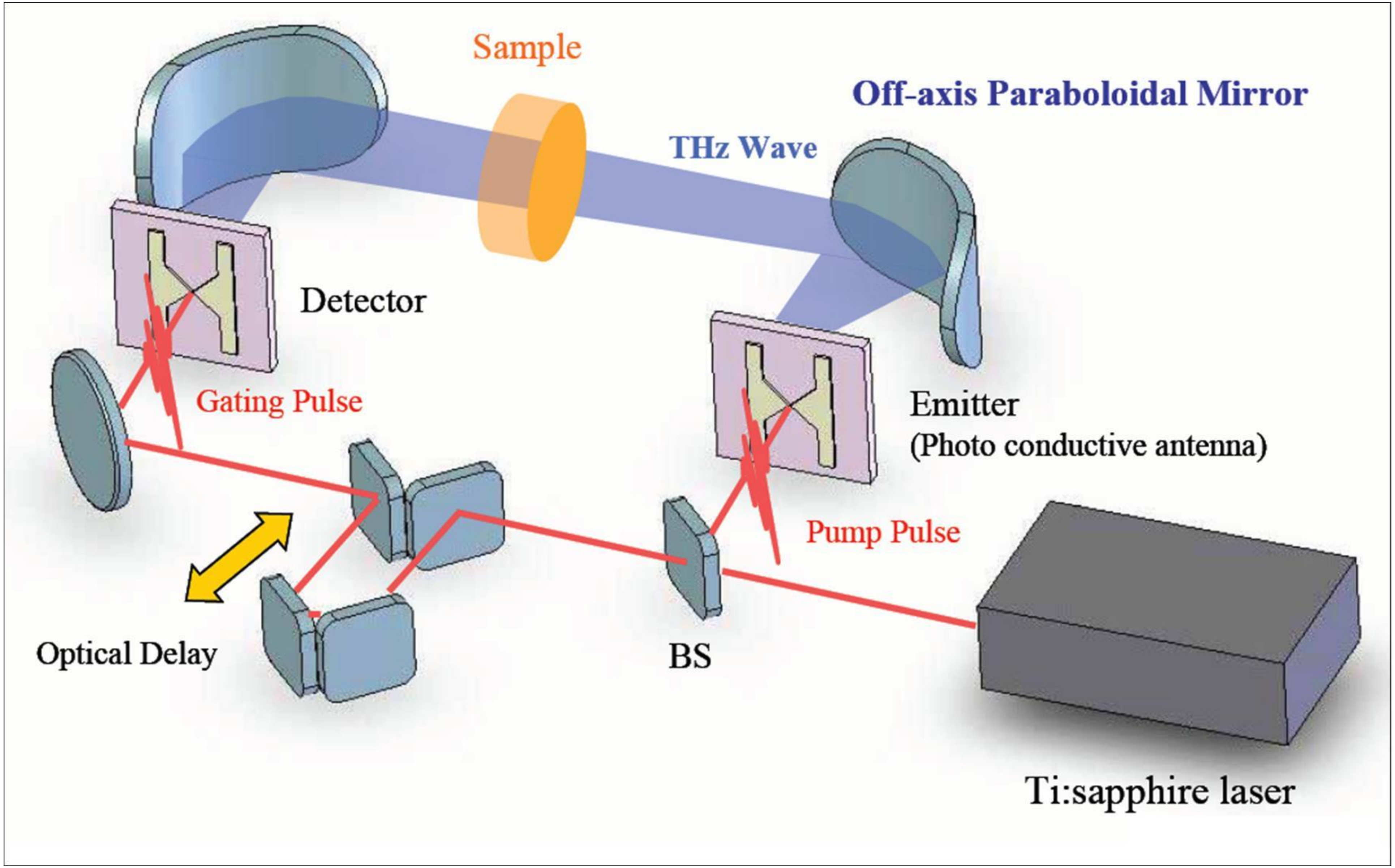
Auston switch



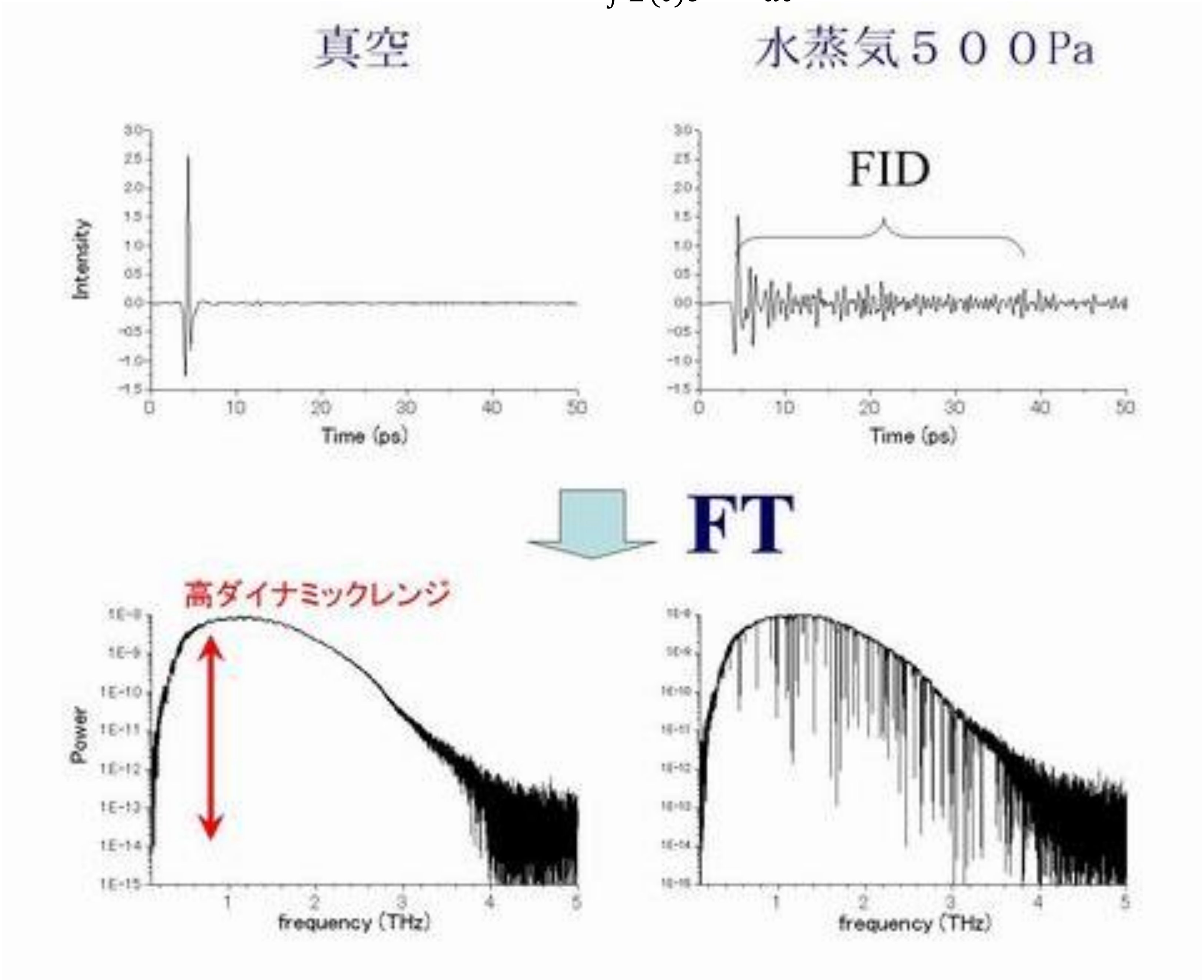
$$\tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t)dt$$

Matthias C. Hoffmann et al., Journal of Physics D: Applied Physics 44, 8 (2011) 83001

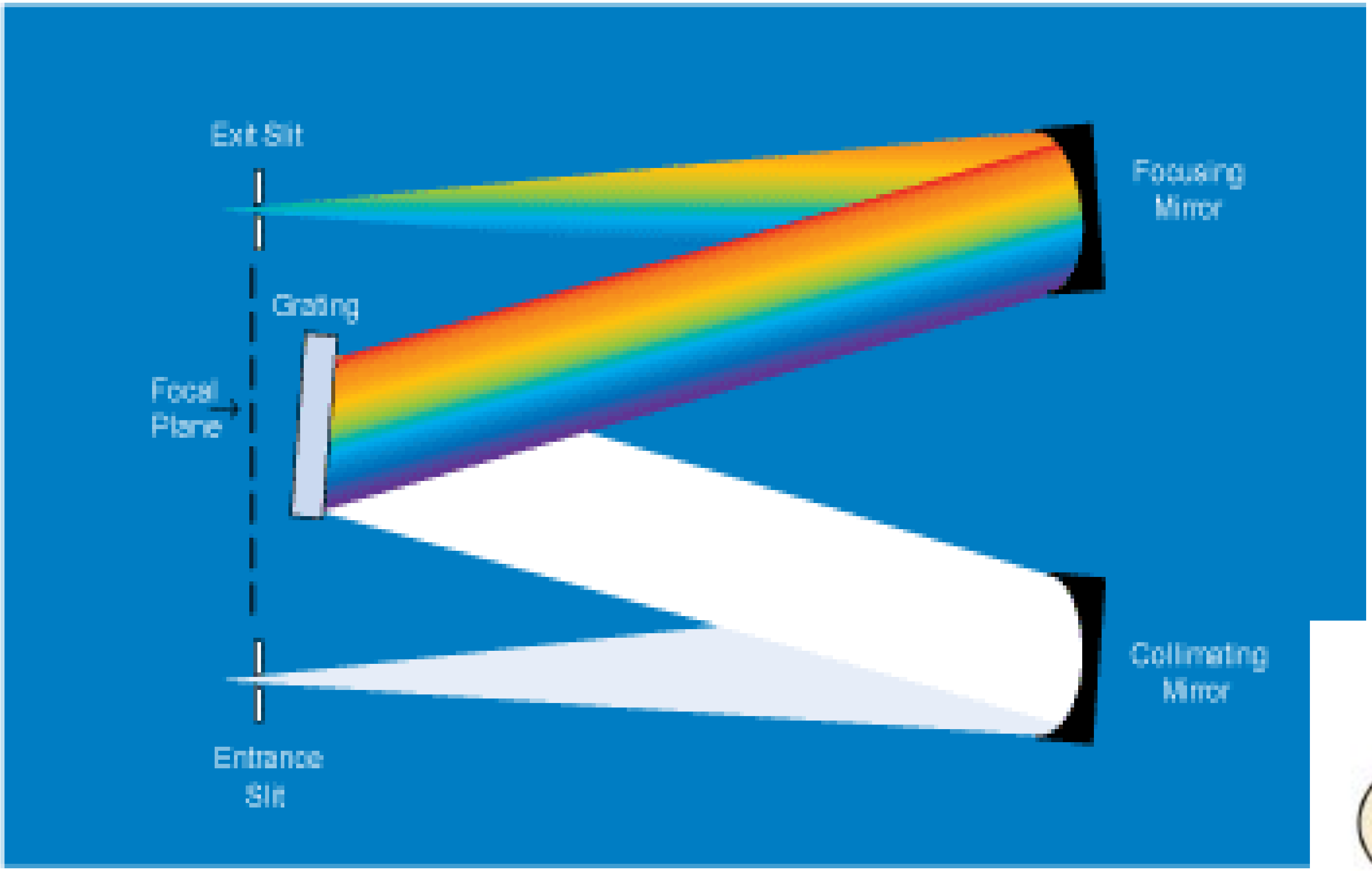
THz time domain spectroscopy

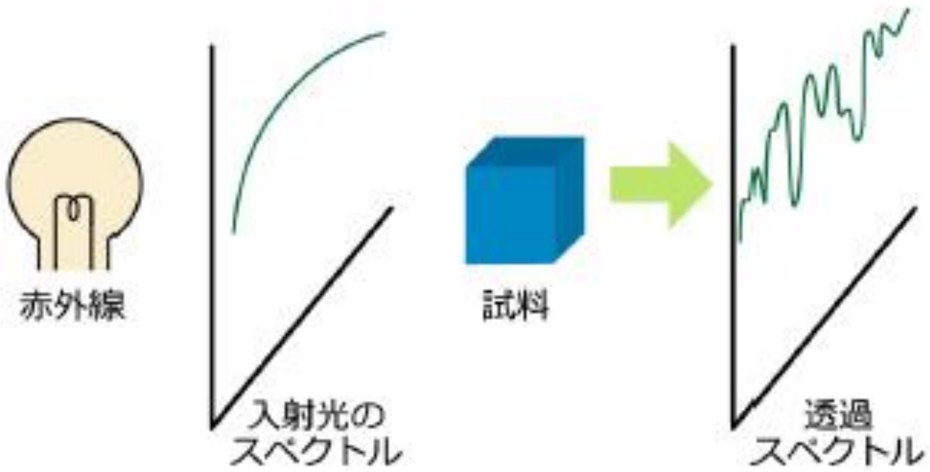


透過電場α(t)E(t)の観測　=>　$S(\omega)=\frac{\int \alpha(t)E(t)e^{-i\omega t}dt}{\int E(t)e^{-i\omega t}dt}$

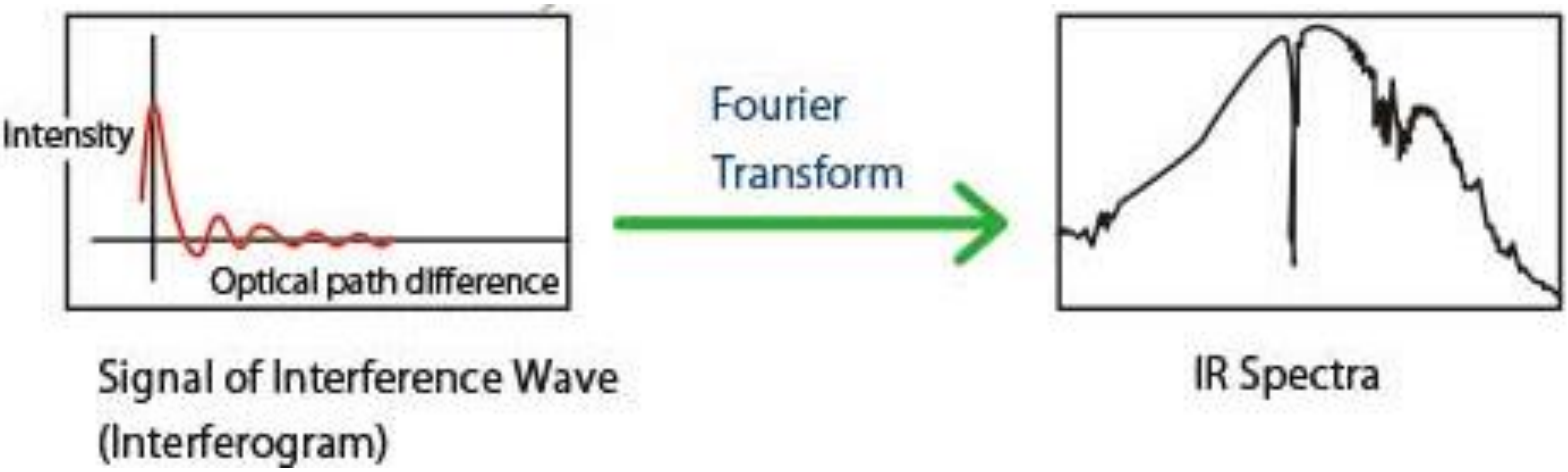


回折格子を使った分光器





Temporal domain spectroscopy



Temporal domain spectroscopy

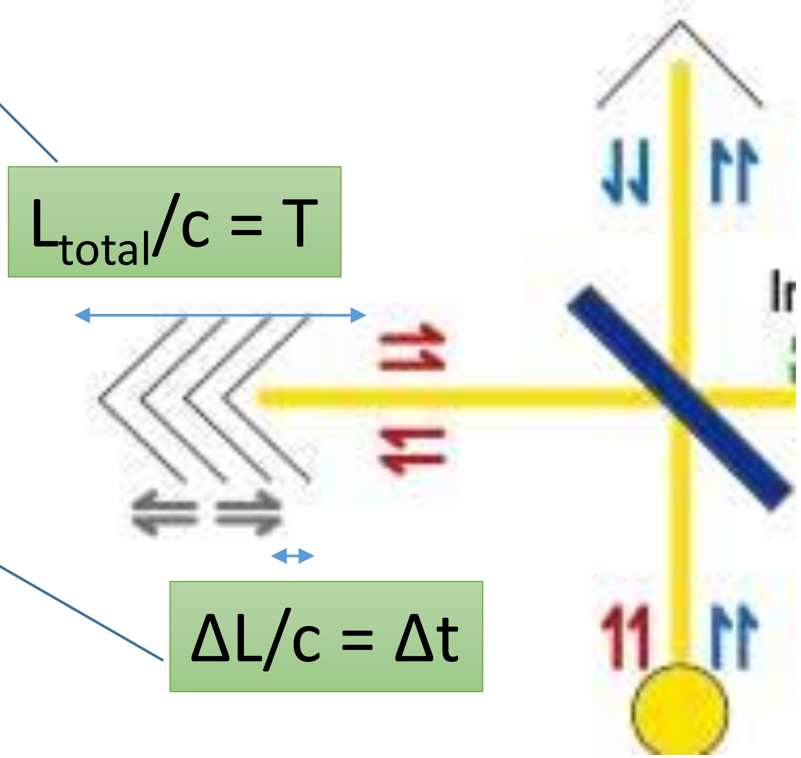
T 観測時間範囲

Δt 観測時間分解能

ω_{max} - ω_{min} スペクトル範囲

$\Delta\omega$ スペクトル分解能

フーリエ変換



例:
干渉計の長さL=1m, 観測ステップ0.01mm

$\Delta\omega$ 〜 300MHz

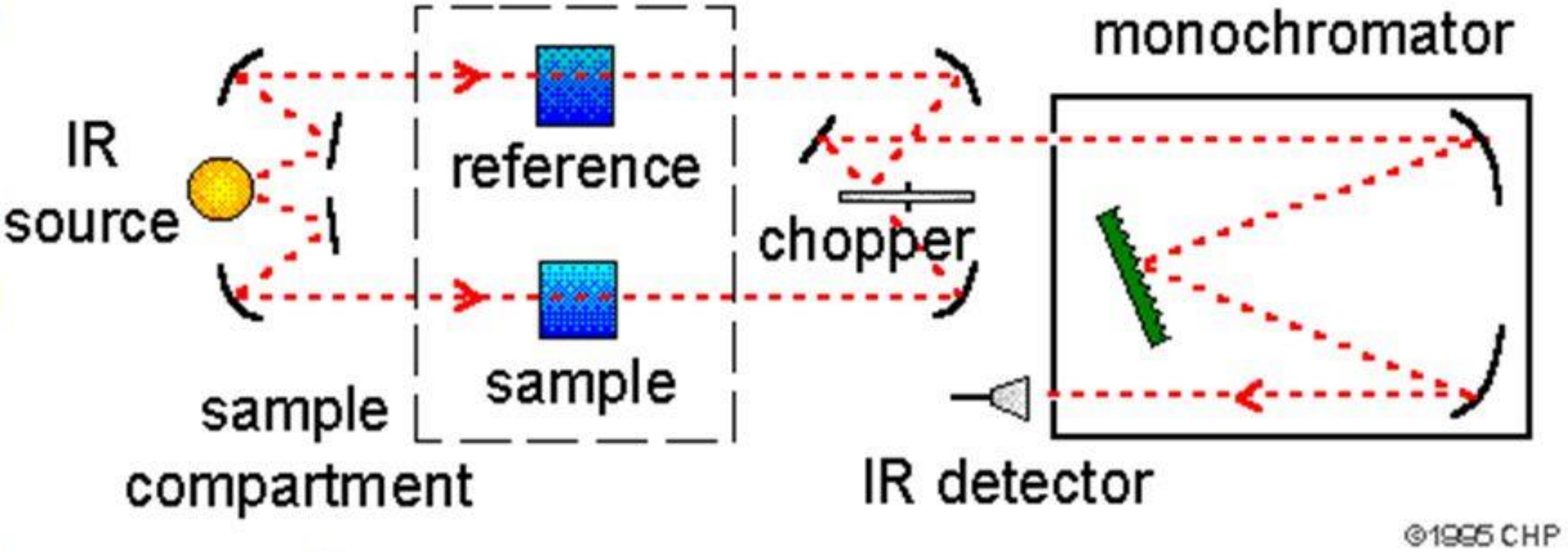
ω_{max} - ω_{min} 〜 30THz

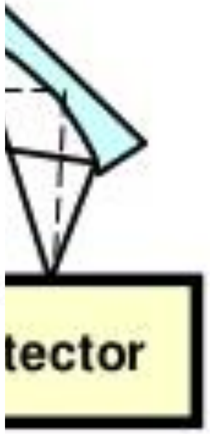
if ω_{center}= 100THz(λ = 3μm), 85 ~ 115 THz

IR Instruments

Infrared Instruments

Schematic of a dispersive (double beam) IR spectrometer





• Sample compartments **before the monochromator** (opposite for UV-vis) to diminish stray/scattered light problems. Possible because IR radiation does not tend to photodecompose compounds, unlike the UV/Vis

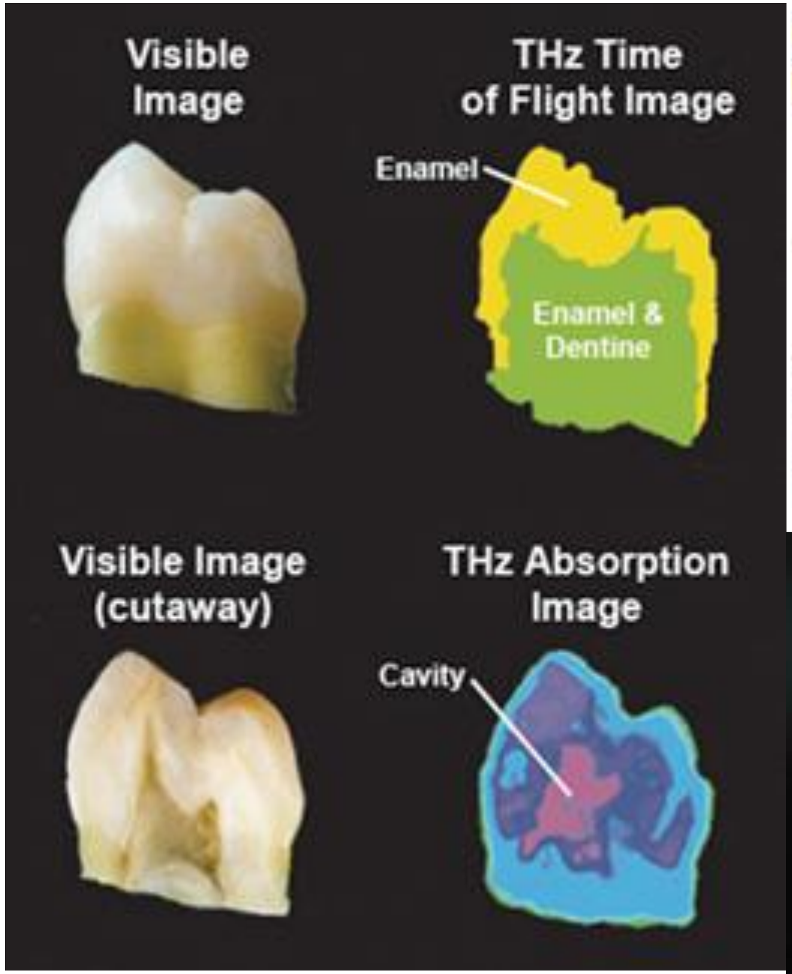
テラヘルツ波

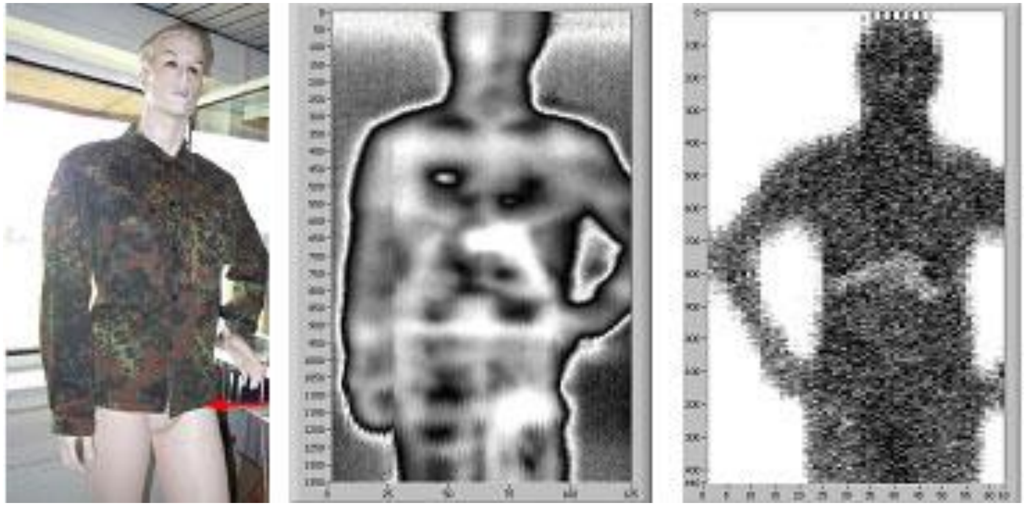
10^9 10^{12} 10^{15} 10^{18}

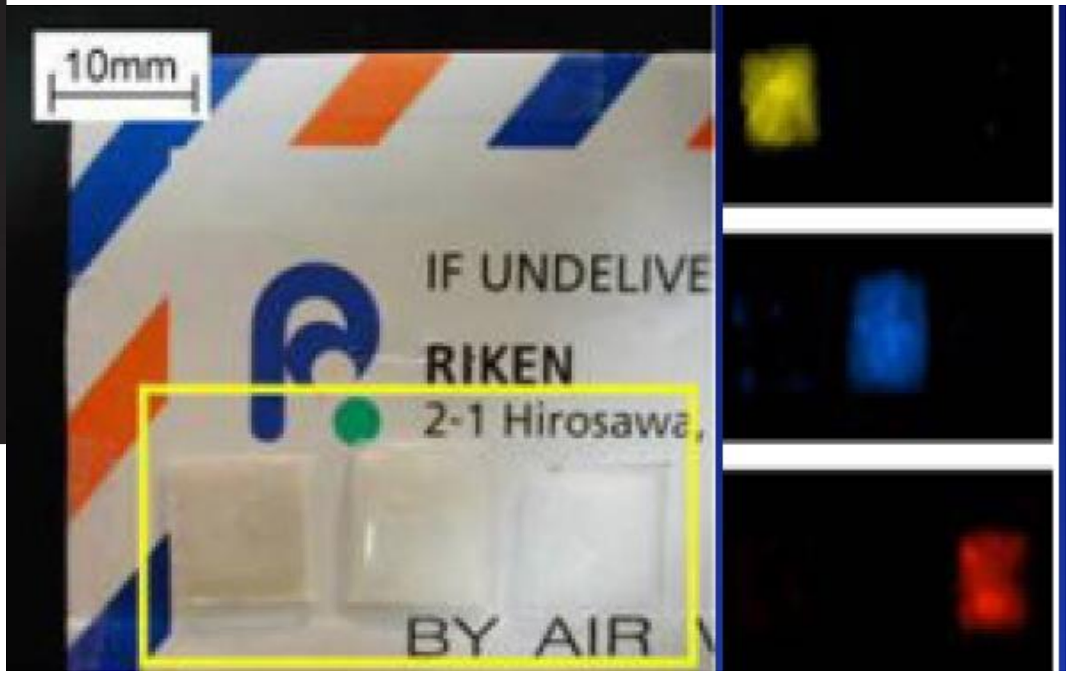
周波数 [Hz]

対応する過程 ⇨ 量子超格子サブレベル　回転振動励起　電子励起　内殻電子励起

温度 ‐‐‐→ 室温　遠赤外線ストーブ　太陽表面　太陽表面　核融合プラズマ


Visible Image
THz Time of Flight Image
Enamel
Enamel & Dentine
Visible Image (cutaway)
THz Absorption Image
Cavity




10mm
IF UNDELIVE
RIKEN
2-1 Hirosawa,
BY AIR

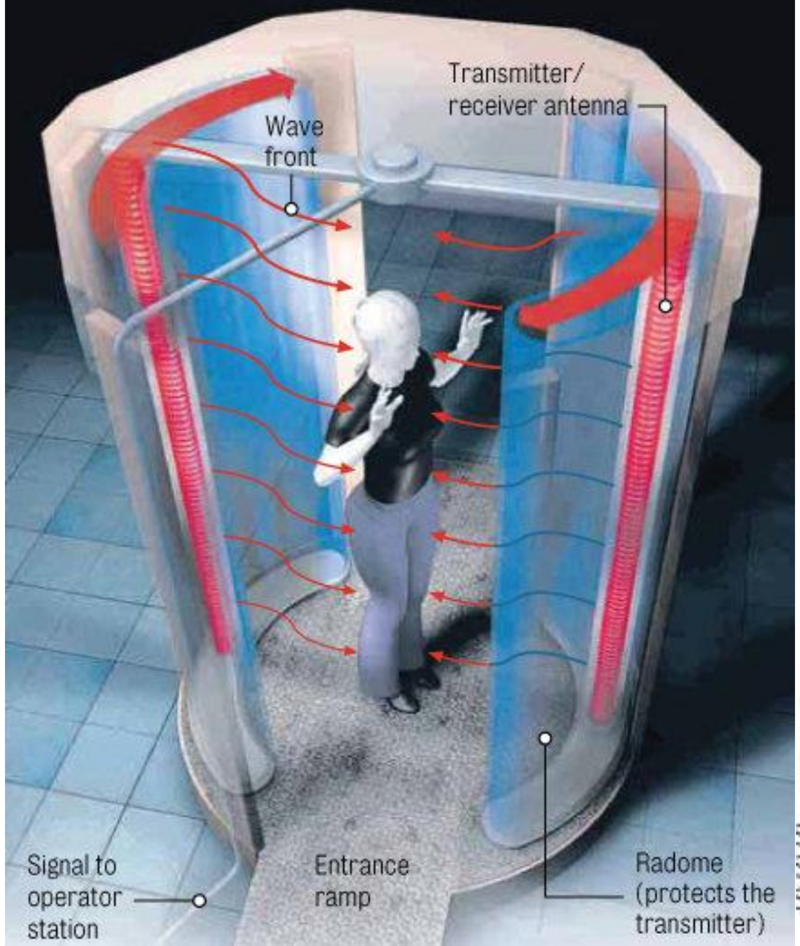
MILLIMETER–WAVE IMAGING

The equipment scans a passenger's entire body and creates a three-dimensional image, minus the passenger's clothing.

HOW IT WORKS:

1 The passenger steps inside the scanner and stands with arms raised.

2 Two antennas quickly rotate around the passenger while projecting beams of radio-frequency energy.



3 Energy projected by the system is one ten-thousandth that of a cell phone.

4 The waves penetrate clothing and reflect off the person and any concealed objects.

Sources: Transportation Security Administration; Scientific American

JONATHAN MORENO, THOMAS McKAY/THE DENVER POST
ABOVE PHOTOS: L3 COMMUNICATIONS / THE ASSOCIATED PRESS